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TECHNICAL REPORT

Methods for calculating the main static performance indicators of transducers and transmitters

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METHODS FOR CALCULATING THE MAIN STATIC PERFORMANCE INDICATORS OF TRANSDUCERS AND TRANSMITTERS

FOREWORD

The main task of IEC technical committees is to prepare International Standards. However, a technical committee may propose the publication of a technical report when it has collected data of a different kind from that which is normally published as an International Standard, for example "state of the art".

IEC TR 62967, which is a technical report, has been prepared by subcommittee 65B: Measurement and control devices of IEC technical committee 65: Industrial-process measurement, control and automation.

The text of this International Standard is based on the following documents:

Enquiry draft	Report on voting
65B/961/DTR	65B/1016/RVC

Full information on the voting for the approval of this technical report can be found in the report on voting indicated in the above table.

This document has been drafted in accordance with the ISO/IEC Directives, Part 2.

The committee has decided that the contents of this document will remain unchanged until the stability date indicated on the IEC website under "http://webstore.iec.ch" in the data related to the specific document. At this date, the document will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

A bilingual version of this publication may be issued at a later date.

INTRODUCTION

This technical report provides a comprehensive illustration of the methods for calculating the main static performance indicators of transducers, transmitters and similar measuring devices. First of all, in order to avoid any misunderstanding, we would like to review the commonly-accepted definition of transducers and transmitters. Generally speaking, in a measurement field, a transducer is a measuring device which converts the non-electrical quantity to be measured into corresponding electrical quantity, while a transmitter is a kind of transducer which is required to provide a previously-given linear output.

The common-in-use standards [01]-[06][1] listed in the relevant documents to be considered in this report, are useful in evaluating the main static performance indicators of measuring instruments and other similar devices. But the relevant descriptions of calculation methods in standards [01]-[05] are not complete and adequate in many ways. This fact was clearly stated in the Introduction of IEC 61298 [03].

On the whole, these publications [01]-[05] mainly contain relevant technical terms and definitions. Since in essence, they are not standards which are dedicated solely to the calculation of performance indicators, so they contain no or only very simple and inadequate illustrations of the calculation methods. Moreover, as these contents have existed for about tens of years, probably now is the time to make an all-round revision and improvement of them. Since there are many static performance indicators that should be calculated and the calculation methods can form a rather complete system. So it is better to create a separate report or a separate standard.

For the main static performance indicators, the existing relevant IEC standards have only theoretical definitions, but have no specific calculation methods. This does not mean that these methods are too simple to mention. But on the contrary, some of them are too difficult to be used in industry. Therefore, this report puts forward, improves and simplifies the existing relevant calculation methods, may probably serve as a good basis on which to create a new calculation-oriented IEC standard.

The report is intended for use by manufacturers to work out their factory-level test standards, by users to make rigorous acceptance tests and wise applications, and by authorized metrological establishments to verify the measuring device performance indicators of the manufacturers or of the users.

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[1] Numbers in square brackets refer to the Bibliography.

METHODS FOR CALCULATING THE MAIN STATIC PERFORMANCE INDICATORS OF TRANSDUCERS AND TRANSMITTERS

1 Scope

This Technical Report provides guidance on the assurance of reliability data of automation devices. If the source of this data is calculation, guidance is given on how to specify the methods used for this calculation. If the source is through observations, guidance is given on how to describe these observations and their evaluations. If the source is the outcome of laboratory tests, guidance is given on how to specify these tests and the conditions under which they have been carried out.

This document defines the form to present the data.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-300, *International Electrotechnical Vocabulary – Electrical and electronic measurements and measuring instruments*

Part 311: General terms relating to measurements

Part 312: General terms relating to electrical measurements

Part 313: Types of electrical measuring instruments

Part 314: Specific terms according to the type of instrument

IEC 60050-351, *International Electrotechnical Vocabulary – Part 351: Control technology*

IEC 60770-1:1999 *Transmitters for Use in Industrial-process Control Systems – Part 1: Methods for Performance Evaluation*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in IEC 60050-300, IEC 60050-351 and IEC 60770-1:1999, as well as the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- IEC Electropedia: available at http://www.electropedia.org/
- ISO Online browsing platform: available at http://www.iso.org/obp

3.1 Basic terms

3.1.1
static characteristics
relationship of the output of a transducer to its input, when the measurand is at the state of stabilization or very slow variation

Note 1 to entry: There are many performance indicators under the title of static characteristics.

Note 2 to entry: Static performance indicators are applicable only under a given interval of temperature.

3.1.2
static calibration
process in which the static characteristics are obtained under given static conditions

3.1.3 Input terms

3.1.3.1
lower range-value
lowest value of the measurand

3.1.3.2
upper range-value
highest value of the measurand

3.1.3.3
measuring range
measuring region indicated by the upper and lowr range-values of the measurand

3.1.3.4
span
span, also called the full-span input, is the algebraic difference between the upper and lower range-values of the measurand.

3.1.4 Output terms

3.1.4.1
zero-range-value output
output when measurand is at its zero range-value

3.1.4.2
lower-range-value output
output when measurand is at its lowest range-value

3.1.4.3
upper-range-value outpu
output when measurand is at its highest range-value

3.1.4.4
full-span output
algebraic difference between the upper-range-value output and lower-range-value output of a device as defined by its working characteristics

3.1.5
linearity
closeness to which the output-input curve of a transducer approximates a straight line

Note 1 to entry: There should be no contribution of hysteresis and repeatability to linearity.

3.1.6
conformity
closeness to which the output-input curve of a transducer approximates a certain curve

Note 1 to entry: There should be no contribution of hysteresis and repeatability to conformity.

3.1.7
reference characteristics
straight line, curve or equation which is used as a reference or a contrast

Note 1 to entry: Under a certain application situation, reference characteristics can be accepted as the true characteristics of a transducer.

Note 2 to entry: In this Technical report, reference characteristics are mainly used in the calculation of linearities, conformities, and linearity (conformity) plus hysterresis.

3.1.8
working characteristics
output-input equation or curve, which is adopted as the true characteristics of a transducer

Note 1 to entry: Working Characteristics has taken into consideration the combined contribution of linearity (conformity), hysteresis and repeatability.

3.1.9
utilization characteristics
relationship of the measurand to the output of a transducer

3.1.10
linear transducer
kind of transducers whose working characteristics are linear

3.1.11
non-linear transducer
kind of transducers whose working characteristics are non-linear

3.2 Static calibration characteristics

3.2.1
up-travel actual average characteristics
curve connecting all the arithmetic average points of a group of measured data at all the calibration points in the up-travel

3.2.2
down-travel actual average characteristics
curve connecting all the arithmetic average points of a group of measured data at all the calibration points in the down-travel

3.2.3
up-travel and down-travel actual average characteristics
curve connecting all the arithmetic average points of a group of measured data at all the calibration points in the up- and down-travel

Note 1 to entry: It is also called the actual average characteristics (or curve) of a transducer.

3.3 Definitions of static performance indicators

3.3.1
resolution
smallest change in input that can cause observable change in output in the whole input span

3.3.2
sensitivity
ratio of output change to its corresponding input change

3.3.3
hysteresis
for the same input and in the whole input span, difference between the values of the down-travel actual average characteristics and the up-travel actual average characteristics

3.3.4
repeatability
for a short time interval and in the same working condition, degree of scatterance of a group of readings obtained when the input is approaching the same measuring point in the same direction for a number of test cycles

3.3.5
linearity
maximum deviation of the actual average characteristics (curve) from the reference straight line

Note 1 to entry: Linearity is expressed as a percentage of full-span output.

Note 2 to entry: According to different reference straight lines taken, there are different kinds of linearities, with the following as the main ones.

Note 3 to entry: When expressed simply as linearity, it is assumed to be independent linearity.

Note 4 to entry: The choice of linearities depends on the application situations of transducers.

3.3.5.1
absolute linearity
also called theoretical linearity, it is calculated from the reference straight line or theoretical straight line that can be specified before the calibration test is made

Note 1 to entry: Absolute linearity actually shows the linearity accuracy of a transducer and is quite different from all the linearities that follow.

Note 2 to entry: Absolute linearity is exclusively used in transmitter applications.

3.3.5.2
terminal-based linearity
linearity calculated from the terminal-based straight line that is taken as the reference straight line

Note 1 to entry: Terminal-based straight line coincides with the actual average characteristics (curve) at its upper and lower limits.

Note 2 to entry: Terminal-based Linearity is easy to calculate, but its value is rather conserved.

3.3.5.3
shifted terminal-based linearity
linearity calculated from the shifted terminal-based straight line that is taken as the reference straight line

Note 1 to entry: The shifted terminal-based straight line has the same slope as the terminal-based straight line and can minimize its maximum deviation from the actual average characteristics (curve) by parallel shifting.

Note 2 to entry: If the device under test has a C-shaped actual average characteristics (curve), the shifted terminal-based straight line will become the best straight line, or best line in short.

3.3.5.4
zero-based linearity
linearity calculated from the zero-based straight line that is taken as the reference straight line

Note 1 to entry: Zero-based straight line goes through the theoretical zero point and can minimize its maximum deviation from the actual average characteristics (curve) by changing its slope.

Note 2 to entry: Sometimes zero-based straight line is also called the forced zero-intersecting best straight line.

3.3.5.5
front-terminal-based linearity
linearity calculated from the front-terminal-based straight line that is taken as the reference straight line

Note 1 to entry: The front-terminal-based straight line goes through the front end of the actual average characteristics (curve) and can minimize its maximum deviation from the actual average characteristics (curve) by changing its slope.

Note 2 to entry: Sometimes and in some references, the front-terminal-based straight line is also called the zero-based straight line.

3.3.5.6
independent linearity
linearity calculated from the best straight line that is taken as the reference straight line

Note 1 to entry: The best straight line is a straight-line midway between the two parallel straight lines closest together and enclosing the actual average characteristics (curve).

Note 2 to entry: The best straight line can minimize its maximum deviation from the actual average characteristics (curve).

3.3.5.7
least-squares linearity
linearity calculated from the least-squares straight line which is adopted as the reference straight line

Note 1 to entry: The least-squares straight line can guarantee that, the sum of the squares of the deviations of the actual average characteristics (curve) from it, is a minimum.

3.3.6
conformity
maximum deviation of the actual average characteristics (curve) from the reference curve

Note 1 to entry: Conformity is expressed as a percentage of full-span output.

Note 2 to entry: According to different reference curves taken, there are different kinds of conformities, with the following as the main ones.

Note 3 to entry: The reference curve is usually in the form of an algebraic polynomial of a certain degree.

Note 4 to entry: When expressed simply as conformity, it is assumed to be independent conformity.

Note 5 to entry: The choice of conformities depends on the application situations of transducers.

3.3.7
absolute conformity
also called theoretical conformity, it is calculated from the reference curve or theoretical curve that can be specified before the calibration test is made

Note 1 to entry: Absolute conformity actually shows the conformity accuracy of a transducer and is quite different from all the conformities that follow.

Note 2 to entry: The reference curve should be specified according to the application requirement of the transducer.

3.3.7.1
terminal-based conformity
conformity calculated from the terminal-based curve that is taken as the reference curve

Note 1 to entry: Terminal-based curve coincides with the actual average characteristics (curve) at its upper and lower limits and can minimize its maximum deviation from the actual average characteristics (curve).

3.3.7.2
zero-based conformity
conformity calculated from the zero-based curve that is taken as the reference curve

Note 1 to entry: Zero-based curve goes through the theoretical zero point and can minimize its maximum deviation from the actual average characteristics (curve).

Note 2 to entry: Sometimes Zero-based curve is also called the forced zero-intersecting best curve.

3.3.7.3
front terminal-based conformity
linearity calculated from the front terminal-based curve that is taken as the reference curve

Note 1 to entry: The front terminal-based curve goes through the front end of the actual average characteristics (curve) and can minimize its maximum deviation from the actual average characteristics (curve).

Note 2 to entry: Sometimes and in some references, the front terminal-based curve is also called the zero-based curve.

3.3.7.4
independent conformity
conformity calculated from the best curve that is taken as the reference curve

Note 1 to entry: The best curve can minimize its maximum deviation from the actual average characteristics (curve).

3.3.7.5
least-squares conformity
conformity calculated from the Least-squares curve that is taken as the reference curve

Note 1 to entry: The least-squares curve can guarantee that the sum of the squares of the deviations of the actual average characteristics (curve) from it is a minimum.

3.3.8
combined linearity (conformity) and hysteresis
extreme value of the systematic error of a transducer

Note 1 to entry: This performance indicator shows the combined contribution, but not the pure addition, of linearity (conformity) and hysteresis.

3.3.9
uncertainty
result of an evaluation that shows a zone in which the true values of the measurand lie under specified operating conditions

Note 1 to entry: It is a parameter that can reasonably show the scatterance of the values of the measurand, and also a parameter connecting with the measurement result.

Note 2 to entry: Uncertainty can more reasonably show the real picture of the total result of a measurement, both qualitatively and quantitatively.

3.3.10
total uncertainty
combined linearity (conformity), hysteresis and repeatability, also called the primary uncertainty, that is obtained from the static calibration under specified conditions and calculated by using specified calculating methods based on the general principle of measurement uncertainty

Note 1 to entry: In this Technical report, total uncertainty is the result of the combined contribution, but not the pure addition, of linearity (conformity), hysteresis and repeatability. This performance indicator is also called the reference accuracy.

3.3.11
zero drift
undesired change in zero-range-value output over a specified period of time

Note 1 to entry: It is usually expressed as a percentage of full-span output.

3.3.12
drift of upper-range-value output
undesired change in upper-range-value output over a specified period of time

Note 1 to entry: It is usually expressed as a percentage of full-span output.

Note 2 to entry: For load cells sometimes this performance indicator is also called creep.

Note 3 to entry: If the specified period of time is very long, for example, several months or years, this performance indicator is usually called long-term stability.

3.3.13
thermal zero shift
undesired change in zero-range-value output with environment temperature

Note 1 to entry: It is usually expressed as a percentage of full-span output per unit temperature.

3.3.14
thermal shift of upper-range-value output
undesired change in upper-range-value output with environment temperature

Note 1 to entry: It is usually expressed as a percentage of full-span output per unit temperature.

4 Methods for calculating individual static performance indicators

4.1 Establishment of static calibration characteristics

4.1.1 General requirements for static calibration

The requirement for the environment and the calibration operation will be specified according to the type and accuracy (uncertainty) degree of the device under calibration by corresponding products manufacturing standards.

The calibration system should include devices for generating standard inputs, excitation power supply, measuring instruments. The accuracy (uncertainty) degree of the calibration system, in general, should be about 3 times higher than that of the device under calibration. The exact accuracy (uncertainty) degree required will be specified by the relevant factory or manufacturing standards.

Static calibration should be carried out in the whole span of the measurand. The measuring points should usually include points at the zero value and the maximum value of the measurand. Equally-spaced m = 5~11 measuring points, and n = 3~5 calibration cycles, are desirable. The data obtained from the calibration should contain as few suspect and unreasonable data points as possible (see Annex E).

Prior to the formal calibration cycle, the device under calibration should be exercised by a number of full range traverses in each direction. At each measuring point, the input is held steady until the output becomes stabilized at its desired final value. Tapping or vibrating the device under calibration is not allowed unless the performance characteristics under study requires such action.

4.1.2 The calculation of static calibration characteristics

4.1.2.1 Up-travel actual average characteristics ($\overline{y}_{u,i}$)

$$\overline{y}_{u,i} = \frac{1}{n}\sum_{j=1}^{n} y_{u,ij} \tag{1}$$

Where $\bar{y}_{u,i}$ is the arithmetic average of a group of readings taken at the ith measuring point in the up-travel and $y_{u,ij}$ is the jth reading at the ith measuring point in the up-travel (I = 1~m, j = 1~n).

4.1.2.2 Down-travel actual average characteristics ($\bar{y}_{d,i}$)

$$\bar{y}_{d,i} = \frac{1}{n}\sum_{j=1}^{n} y_{d,ij} \tag{2}$$

where $\bar{y}_{d,i}$ is the arithmetic average of a group of readings taken at the ith measuring point in the down-travel and $y_{d,ij}$ is the jth reading at the ith measuring point in the down-travel (I = 1~m, j = 1~n).

4.1.2.3 Up- and down-travel actual average characteristics ($\bar{y}_i$)

The following formula is used:

$$\bar{y}_i = \frac{1}{2}(\bar{y}_{u,i} + \bar{y}_{d,i}) \tag{3}$$

4.2 Span (x_{FS})

The following formula is used:

$$x_{FS} = x_{max} - x_{min} \tag{4}$$

where x_{max} is the upper range-value of the measurand and x_{min} is the lower range-value of the measurand.

4.3 Full-span output (YFS)

The following formula is used:

$$Y_{FS} = Y_{max} - Y_{min} \tag{5}$$

where Y_{max} is the upper-range-value Output as defined by the working characteristics and Y_{min} is the lower-range-value Output as defined by the working characteristics.

NOTE 1 Capital Y stands for outputs calculated from the characteristics which is derived from curve fitting (for example, working characteristics, reference characteristics), and given characteristics (for example, those for transmitters); while small y stands for the actual outputs (readings).

NOTE 2 In situations where individual performance indicator is calculated, full-span output can be calculated from the reference characteristics, instead of the working characteristics.

NOTE 3 For devices having linear or C-shaped characteristics, it is also possible to use $Y_{FS} = Y(x_{max}) - Y(x_{min})$.

NOTE 4 In some applications where no strict requirement is needed, it is allowed to calculate the full-span output by the formula ($y_{FS} = y_{max} - y_{min}$).

4.4 Resolution (Rx)

The following formula is used:

$$R_x = \text{Max} \mid \Delta x_{i,\ min} \mid \quad (6)$$

where $\Delta x_{i,\ min}$ is the smallest change in input at the ith measuring point that can cause observable change in output and Max | $\Delta x_{i,\ min}$ | is the largest $\Delta x_{i,\ min}$, effective over the whole input span, namely the smallest change in input that can cause observable change in output everywhere over the whole input span.

NOTE Dead band and threshold are usually regarded as the resolution at zero range-value.

4.5 Sensitivity (Si)

The sensitivity at the ith measuring point is calculated by the formula:

$$Si = \lim_{\Delta x_i \to 0} \left(\frac{\Delta Y_i}{\Delta x_i} \right) = \frac{dY_i}{dx_i} \quad (7)$$

where Δx_i is the change in input at the ith measuring point and ΔY_i is the change in output at the ith measuring point which is caused by Δx_i.

For linear transducers, sensitivity is a constant:

$$S = \frac{Y_{max} - Y_{min}}{x_{max} - x_{min}} \quad (8)$$

4.6 Hysteresis (ξH)

The following formula is used:

$$\xi_H = \frac{\left| Y_{H,max} \right|}{Y_{FS}} \times 100\% \quad (9)$$

where $\Delta Y_{H,max}$ = Max $\left|\overline{y}_{d,i} - \overline{y}_{u,i}\right| \overline{y}_{d,i}$ is down-travel actual average characteristics and is up-travel actual average characteristics.

NOTE The hysteresis in this report includes both hysteresis and dead band, a practice similar to that used in IEC 60770 and IEC 61298.

4.7 Repeatability (ξR)

4.7.1 Calculating methods

The repeatability of a transducer is the extreme value of its random error. The repeatability at a certain measuring point can be regarded as the extreme value of the sample standard deviation, at a certain confidence level, of a group of readings taken at the point for n test cycles. Repeatability is expressed as a percentage of full-span output. The largest repeatability from the repeatability at all measuring points is chosen as the repeatability of the transducer. Calculating formula is as follows:

$$\xi_R = \frac{cs_{max}}{Y_F} \times 100\% \quad (10)$$

where

c is coverage factor, c = t 0,95 and s_{max} is the largest standard deviation, which is chosen from the 2m sample standard deviations of m measuring points.

NOTE The choice of the largest standard deviation smax conforms with that of IEC 60770 and IEC 61298.

4.7.2 Determination of coverage factor

For a small sample (n = 3~5 cycles), t-distribution is more reasonable than normal distribution. This report chooses t-distribution and adopts a coverage factor c = t 0,95 (for a confidence level of 95 %). In case of necessity, c may not be taken as t 0,95, but this should be clearly stated. Table 1 shows the relationship of coverage factor c = t 0,95 in relation to the number of test cycles n.

Table 1 – Form to present reliability data with its data types

(n)	2	3	4	5	6	7	8	9	10
t 0,95	12,71	4,303	3,182	2,776	2,571	2,447	2,365	2,306	2,262

4.7.3 Calculation of sample standard deviations

By using Bessel's formula, we can obtain 2m sample standard deviations:

$$s_{u,i} = \sqrt{\frac{\sum_{j=1}^{n}(y_{u,ij} - \bar{y}_{u,i})^2}{n-1}} \tag{11}$$

$$s_{d,i} = \sqrt{\frac{\sum_{j=1}^{n}(y_{d,ij} - \bar{y}_{d,i})^2}{n-1}} \tag{12}$$

where $s_{u,i}$ is the sample standard deviation at the ith measuring point in the up-travel, $s_{d,i}$ is the sample standard deviation at the ith measuring point in the down-travel, $\bar{y}_{u,i}$ is the arithmetic average of a group of readings at the ith measuring point in the up-travel, $y_{u,ij}$ is the jth reading at the ith measuring point in the up-travel (i = 1~m; j = 1~n), $\bar{y}_{d,i}$ is the arithmetic average of a group of readings at the ith measuring point in the down-travel, $y_{d,ij}$ is the jth reading at the ith measuring point in the down-travel (i = 1~m; j = 1~n), m is the number of measuring points, n is the number of test cycles.

4.8 Linearity (ξL)

4.8.1 The general formula for calculating linearity

$$\xi_L = \frac{\Delta Y_{L,max}}{Y_{FS}} \times 100\% \tag{13}$$

where

$\Delta Y_{L,max} = \mathrm{Max}\left(\bar{y}_i - Y_i\right)$ is the maximum deviation of the actual average characteristics from the reference characteristics;

$\bar{y}_i$ is the actual average characteristics at the ith measuring point;

Y_i is the reference characteristics at the ith measuring point, Y_{FS} is the full-span output.

NOTE 1 Hints for calculating $\triangle Y_{L,max}$:

1) By using the actual average characteristics ($\bar{y}_i$), and with the best straight line as the reference characteristics, we can obtain independent linearity.

1) By using the actual average characteristics ($\bar{y}_i$), and with the working characteristics as the reference characteristics, we can obtain a kind of absolute linearity. But, this kind of absolute linearity probably includes components of hysteresis and repeatability, and therefore is, in a strict sense, not an indication of pure linearity.

NOTE 2 Sometimes and in case of necessity, it is permissible to calculate linearity simply from a group of calibration readings, instead of from ($\bar{y}_i$). As the calculation of some linearity is rather complicated, it is recommended that the calculation should be done with computer software.

4.8.2 Absolute linearity (ξL,ab)

Pre-specify the equation of the reference straight line and use Formula (13).

NOTE 1 Among the several linearities in this report, and for the same linearity value, absolute linearity is the most difficult to obtain.

NOTE 2 If interchangeability of characteristics is needed in a transducer, it is desirable to adopt absolute linearity.

NOTE 3 As transmitters have pre-specified characteristics, and therefore they adopt absolute linearity.

NOTE 4 For transducers and transmitters having digital displays, the displayed readings and the measurand have a pre-specified equation Y_{ab} = x, therefore absolute linearity is adopted.

4.8.3 Terminal-based Linearity (ξL,te)

The principle of calculating method is shown in Figure 1 and calculating formula is shown in Formula (13).

The formula for the terminal-based straight line is as follows:

$$Y_{te} = y_{min} - \frac{\bar{y}_{max} - \bar{y}_{min}}{x_{max} - x_{min}} + \frac{\bar{y}_{max} - \bar{y}_{min}}{x_{max} - x_{min}} \qquad (14)$$

or

$$Y_{te} = a + bx \qquad (15)$$

where

$b = \frac{\bar{y}_{max} - \bar{y}_{min}}{x_{max} - x_{min}}$ is the slope of the terminal-based straight line;

$a = \bar{y}_{min} - bx_{min}$ is the intercept of the terminal-based straight line;

$\bar{y}_{max}$, $\bar{y}_{min}$ is the maximum and minimum values of the actual average characteristics;

x_{max}, x_{min} is the maximum and minimum values of the measurand.

NOTE 1 Terminal-based straight line is easy to obtain, and terminal-based linearity is easy to determine by using an electric bridge circuit.

NOTE 2 As compared with values of other linearities, the value of terminal-based linearity is rather large.

4.8.4 Shifted-terminal-based Linearity ($\xi L,s,te$)

Calculating formula is shown in Formula (13).

The slope of the shifted-terminal-based straight line is the same as that of the terminal-based straight line. But it should be parallel-shifted so as to minimize the maximum deviations.

NOTE 1 In situations where no strict requirement is needed, shifted-terminal-based linearity may be used as a substitute for independent linearity.

NOTE 2 An example of the application of the shifted-terminal-based linearity is shown in in A.2.2.1 of Annex A.

4.8.5 Zero-based linearity ($\xi L,ze$)

The principle of calculating method is shown in Figure 2 and calculating formula is shown in Formula (13). According to the definition of zero-based straight line we can work out the following equation:

$$Y_{ze} = bx \qquad (16)$$

where

b is the slope of the zero-based straight line.

Zero-based straight line can be calculated as a straight line that goes through the theoretical zero point (x = 0, y = 0) and the centre of gravity of the two points which have minimal positive and negative deviations respectively.

The calculating method for the zero-based straight line is shown in Clause A.1.

NOTE Zero-based straight line is simple in equation form and is therefore easy in application.

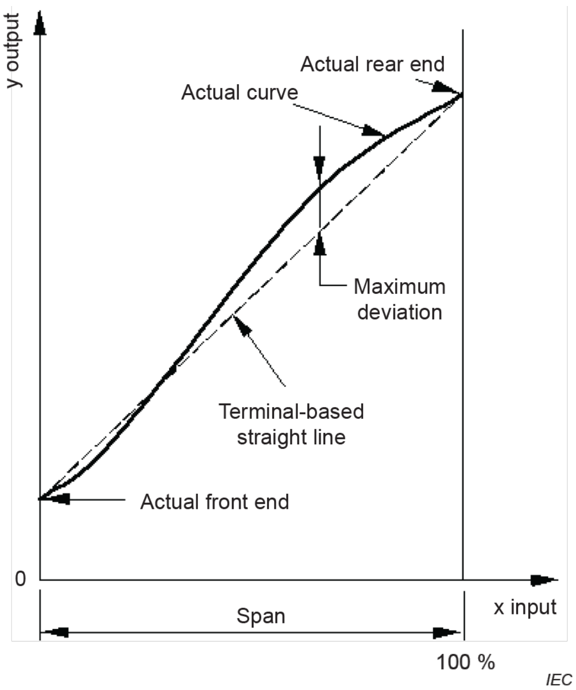


Figure 1 – Terminal-based Linearity

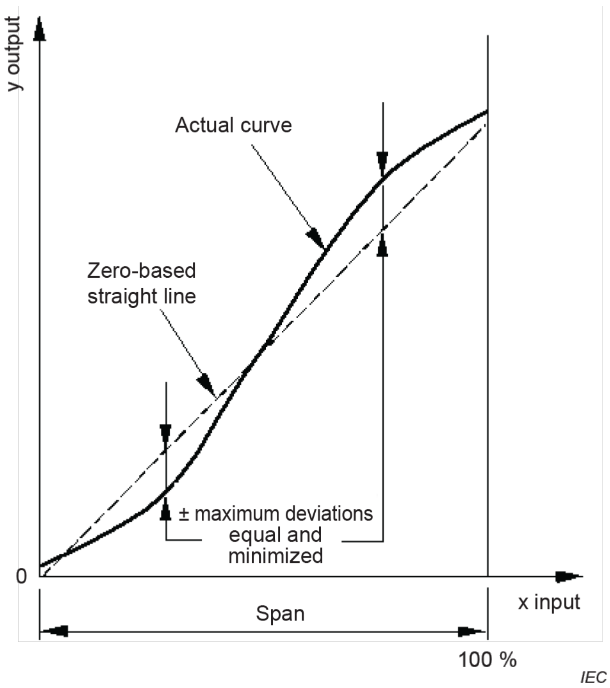


Figure 2 – Zero-based Linearity

4.8.6 Front-terminal-based Linearity (ξL,f,te)

The principle of calculating method is shown in Figure 3 and calculating formula is shown in Formula (13). A similar calculating method for the front-terminal-based straight line is shown in Clause A.1.

NOTE 1 The calculating method for front-terminal-based straight line is similar to that used in calculating zero-based straight line. The only difference lies in that the Front-terminal-based straight line has an intercept whose value is the y ordinate of the front end (with abscissa x being zero) of the actual average characteristics.

NOTE 2 If adjustment is possible in a transducer, front-terminal-based straight line can be turned into a zero-based straight line by parallel-shifting the coordinate system.

NOTE 3 In terms of the linearity value, front-terminal-based linearity is better than zero-based linearity. Front-terminal-based straight line can guarantee that the deviations around the zero input are relatively small.

4.8.7 Independent Linearity (ξL,in)

The principle of calculating method is shown in Figure 4 and the calculating formula is shown in Formula (13). The calculating method for the best straight line is shown in Clause A.2.

NOTE 1 Among all the linearities, independent linearity is the best. For precise evaluation of linearity, it is preferable to use independent linearity.

NOTE 2 If adjustment is possible in a transmitter, it is possible to make the best straight line the pre-specified straight line by slope changing and parallel shifting, so as to obtain the highest absolute linearity.

NOTE 3 As the calculation of independent linearity is rather complicated, it is recommended that the calculation should be done with a computer software.

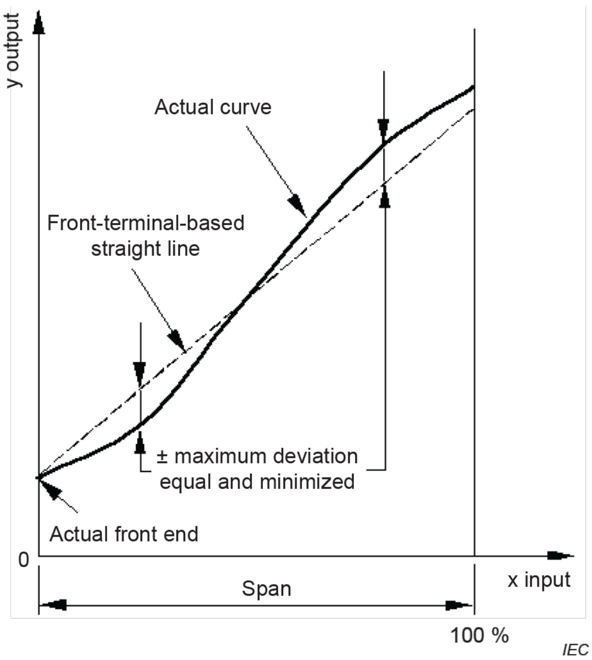


Figure 3 – Front-terminal-based Linearity

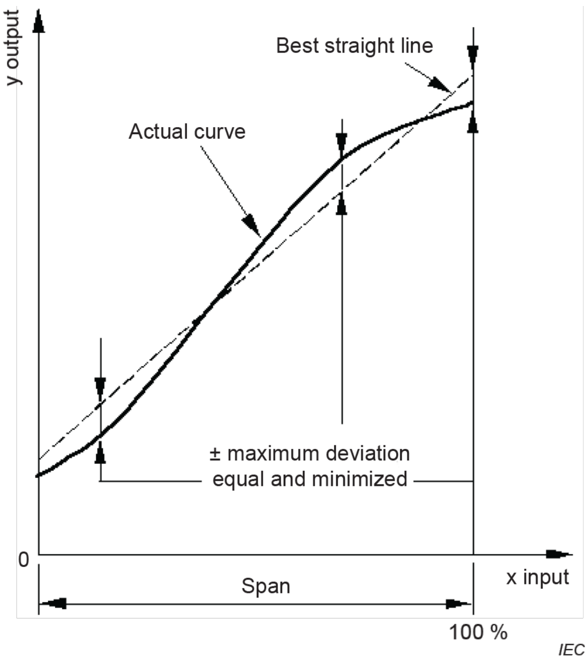


Figure 4 – Independent Linearity

4.8.8 Least-squares Linearity (ξL,ls)

The calculating formula is shown in Formula(13).

The equation for the least-squares straight line is as follows:

$$Y_{ls} = a + bx \qquad (17)$$

where

x is the input of the transducer;

Y_{ls} is the reference output of the transducer;

a,b is the intercept and slope of the least-squares straight line, and they are:

$$a = \frac{\Sigma x_i^2 \cdot \Sigma y_i - \Sigma x_i \cdot \Sigma x_i y_i}{m\Sigma x_i^2 - (\Sigma x_i)^2} \qquad (18)$$

$$b = \frac{m\Sigma x_i y_i - \Sigma x_i \cdot \Sigma y_i}{m\Sigma x_i^2 - (\Sigma x_i)^2} \qquad (19)$$

Where,

$\sum x_i = x_1 + x_2 + \ldots + x_m$;

$\sum y_i = y_1 + y_2 + \ldots + y_m$;

$\sum x_i y_i = x_1 y_1 + x_2 y_2 + \ldots + x_m y_m$;

$\sum x_i^2 = x_1^2 + x_2^2 + \ldots + + x_m^2$;

x_i is the input at the ith measuring point;

y_i is the value of the actual average characteristics at the ith measuring point;

m is the number of measuring points.

NOTE 1 Least-squares straight line cannot guarantee that the maximum deviation is a minimum;

NOTE 2 For reducing deviations, a least-squares straight line can be parallel-shifted, so as to make the maximum positive and negative deviations equal in absolute value;

NOTE 3 In situations where no strict requirement is needed, shifted least-squares straight line may be used as a substitute for the best straight line for independent linearity.

4.9 Conformity (ξC)

Conformity is meaningful only after the form of the fitting functions has been chosen. Moreover, a comparison between the conformities of the different kinds of transducers is feasible only on condition that they have fitting functions of the same form and the same degree. Depending on the different forms and degrees of the fitting curves, there are more kinds of conformities than those of linearities. This document recommends, on the premise that the requirement for conformity has been satisfied, that the use of algebraic polynomials with degrees as low as possible is desirable.

4.9.1 The general formula for calculating conformity

$$\xi C = \frac{\Delta Y_{C,max}}{Y_{FS}} \times 100\% \qquad (20)$$

where

$\Delta Y_{C,max} = \text{Max}\left(\bar{y}_i - Y_i\right)$ is the maximum deviation of the actual average characteristics from the reference characteristics;

$\bar{y}_i$ is the actual average characteristics at the ith measuring point;

Y_i is the reference characteristics at the ith measuring point;

Y_{FS} is the full-span output.

NOTE 1 Hints for calculating $\Delta Y_{L,\ max}$:

1) By using the actual average characteristics ($\bar{y}_i$), and with a best curve as the reference characteristics, we can obtain an independent conformity.

2) By using the actual average characteristics ($\bar{y}_i$), and with the working characteristics as the reference characteristics, we can obtain a kind of absolute conformity. But, this kind of absolute conformity probably includes components of hysteresis and repeatability, and therefore is, in a strict sense, not an indication of pure conformity.

NOTE 2 Sometimes and in case of necessity, it is permissible to calculate conformity simply from a group of calibration readings, instead of from ($\bar{y}_i$).

NOTE 3 There are many kinds of conformities, such as: independent conformity of the first terminal-based conformity of the second degree, zero-based conformity of the third degree, least-squares conformity of the fourth degree.

NOTE 4 As the calculation of conformities, especially of the higher degrees, is rather complicated, it is recognized that the calculation must be done with a computer software.

4.9.2 Absolute conformity (ξC,ab)

Pre-specify the equation of the reference straight line and use Formula (20).

NOTE 1 Among the several conformities in this report, and for the same conformity value, absolute conformity Is the most difficult to obtain.

NOTE 2 If interchangeability of characteristics is needed in a transducer, it is desirable to adopt absolute conformity.

4.9.3 Terminal-based conformity (ξC,te)

The calculating formula is shown in Formula (20). The method for deriving the terminal-based curve is shown in Annex B.

NOTE If smaller deviations are desired around the two ends of the input span, terminal-based curve may be a better choice.

4.9.4 Zero-based conformity (ξC,ze)

The calculating formula is shown in Formula (20). The method for deriving the zero-based curve is shown in Annex B.

NOTE 1 Zero-based curve is simple in equation form and is therefore easy in application.

NOTE 2 A zero-based curve cannot necessarily guarantee that the zero-range-value output is a zero.

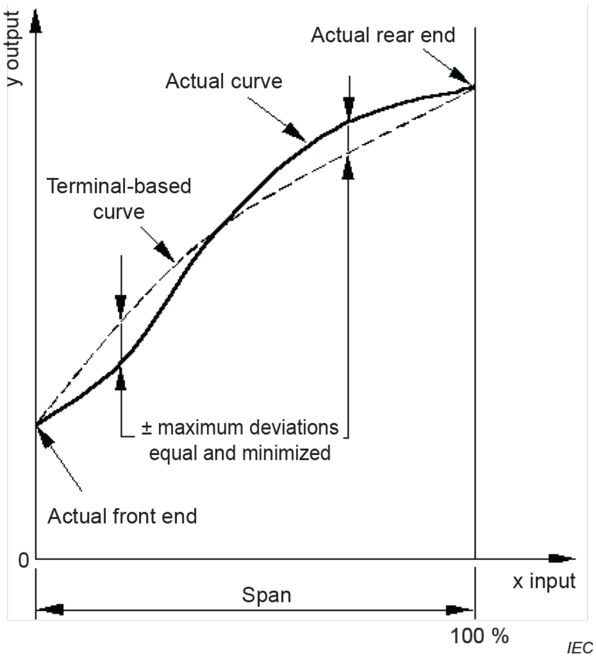


Figure 5 – Terminal-based conformity

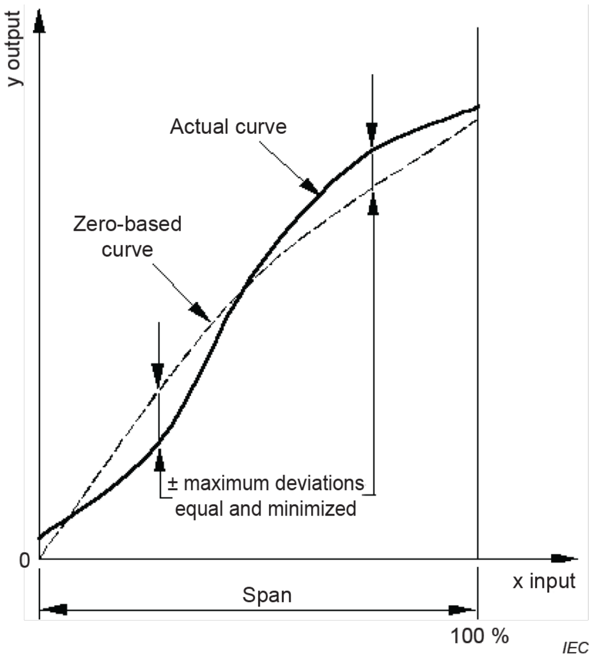


Figure 6 – Zero-based conformity

4.9.5 Front-terminal-based conformity (ξC,f,te)

The calculating formula is shown in Formula (20). The method for deriving the front-terminal-based curve is shown in Annex B.

NOTE 1 If adjustment is possible in a transducer, front-terminal-based curve can be turned into a zero-based curve by parallel-shifting the coordinate system.

NOTE 2 For the same form and the same degree of fitting curve, and in terms of the conformity value, front-terminal-based conformity is better than zero-based conformity. Front-terminal-based curve can guarantee that the deviations around the zero input are relatively small.

4.9.6 Independent conformity (ξC,in)

The calculating formula is shown in Formula (20). The method for deriving the best curve is shown in Annex B.

NOTE 1 For the same form and the same degree of fitting curve, and in terms of the conformity value, independent conformity has the highest conformity among all the conformities.

NOTE 2 If adjustment is possible in a transducer, it is desirable to make the best curve approximate the pre-specified curve, so as to obtain a better absolute conformity.

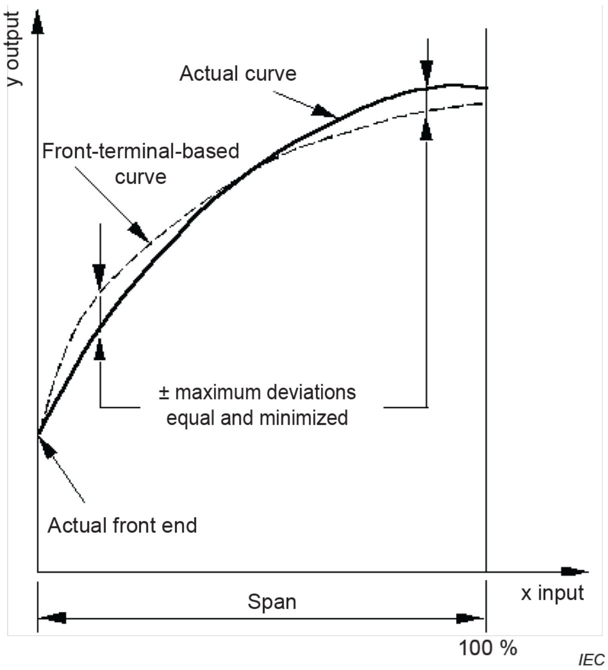


Figure 7 – Front-terminal-based conformity

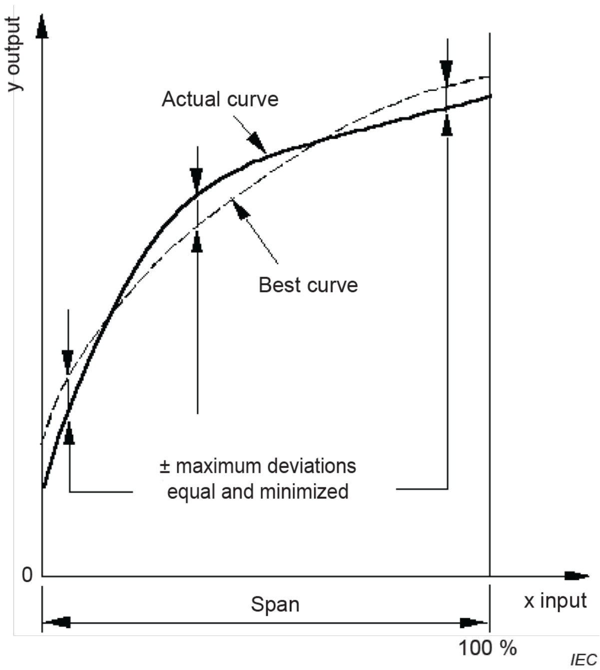


Figure 8 – Independent conformity

4.9.7 Least-squares conformity (ξC,ls)

The calculating formula is shown in Formula (20).

The equation for the least-squares curve expressed in algebraic polynomial is as follows:

$$Y_{ls} = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n \qquad (21)$$

where

x is the input of the transducer;

Y_{ls} is the reference output of the transducer;

$a_0 \ldots a_n$ is the coefficients determining the form and position of the least-squares curve and they can be derived from the curve fitting of the actual average characteristics of a transducer.

NOTE 1 Least-squares curve cannot guarantee that the maximum deviation is a minimum;

NOTE 2 For reducing deviations, a least-squares curve can be parallel-shifted, so as to make the maximum positive and negative deviations equal in absolute value.

NOTE 3 In situations where no strict requirement is needed, shifted least-squares curve may be used as a substitute for the best curve for independent conformity.

NOTE 4 The calculating formula for the least-squares curve of the second degree is shown in B.2.4.2.

4.10 Drift and shift

4.10.1 Zero drift (D0)

The zero drift of a transducer (D0) can be calculated as follows:

$$D_0 = \frac{\Delta y_0}{Y_{FS}} \times 100\ \% \tag{22}$$

$$= \frac{\left| y_{0,max} - y_{0,i} \right|}{Y_{FS}} \times 100\ \%$$

where

$y_{0,i}$ is the initial zero-range-value output;

$y_{0,max}$ is the largest zero-range-value output over the specified period of time;

Y_{FS} is the full-span output (for convenience of calculation, actual full-span output is allowed to be used here).

4.10.2 Drift of upper-range-value output (Du)

The drift of upper-range-value output of a transducer (Du) can be calculated as follows:

$$D_u = \frac{\Delta y_u}{Y_{FS}} \times 100\ \% \tag{23}$$

$$= \frac{\left| y_{u,max} - y_{u,i} \right|}{Y_{FS}} \times 100\ \%$$

where,

$y_{u,i}$ is the initial upper-range-value output;

$y_{u,max}$ is the largest upper-range-value output over the specified period of time;

Y_{FS} is the full-span output (for convenience of calculation, actual full-span output is allowed to be used here).

4.10.3 Thermal zero shift (γ)

The Thermal zero shift of a transducer (γ) can be calculated as follows:

$$= \frac{\left| \bar{y}_{0(T2)} - \bar{y}_{0(T1)} \right|}{Y_{FS(T1)}\left(T_2 - T_1\right)} \times 100\ \%(/) \tag{24}$$

Where

$\bar{y}_{0(T1)}$ is the average zero-range-value output of a transducer at temperature T_1;

$\bar{y}_{0(T2)}$ is the average zero-range-value output of a transducer at temperature T_2;

$Y_{FS(T1)}$ is the full-span output at temperature T_1 (for convenience of calculation, actual full-span output is allowed to be used here).

If the zero-range-value output of a transducer is non-linear with temperature, the interval (T_2 – T_1) should be divided into several sub-intervals, and calculate these of all the sub-intervals by using formula (3-27), and choose the γ which has the largest absolute value.

4.10.4 Thermal shift of upper-range-value output (β)

The Thermal shift of upper-range-value output of a transducer (γ) can be calculated as follows:

$$\beta = \frac{\left|\bar{y}_{FS(T2)} - \bar{y}_{FS(T1)}\right|}{Y_{FS(T1)}\ (T2\ -\ T1)} \times 100\ \%(/) \qquad (25)$$

where

$\bar{y}_{FS(T1)}$ is the average upper-range-value output of a transducer at temperature $T_{1;}$

$\bar{y}_{FS(T2)}$ is the average upper-range-value output of a transducer at temperature $T_{2;}$

$Y_{FS(T1)}$ is the full-span output at temperature T_1 (for convenience of calculation, actual full-span output is allowed to be used here).

If the upper-range-value output of a transducer is non-linear with temperature, the interval ($T_2 - T_1$) should be divided into several sub-intervals, and calculate the β's of all the sub-intervals by using formula (3-28), and choose the β which has the largest absolute value.

5 Methods for calculating combined static performance indicators

Under static conditions, linearity (conformity), hysteresis and repeatability, when put forward individually, are called the individual performance indicators of a transducer. Their different combinations form combined performance indicators. There are no mathematically-determinate relationships between the individual and combined performance indicators.

We will take the linear transducer as an example to illustrate the calculation of combined performance indicators. The calculating principle of the non-linear transducer is similar to that of the linear transducer. In dealing with all the performance indicators, the principle of extreme deviations is used exclusively in their evaluation.

5.1 Combined linearity and hysteresis (Linearity plus hysteresis) ξLH

5.1.1 The general form of calculating formula

$$\xi_{LH} = \pm \frac{\left|\Delta Y_{LH,max}\right|}{Y_{FS}} \times 100\ \% \qquad (26)$$

where

$Y_{LH,max}$ is the maximum deviation of the up-travel actual average characteristics ($\bar{y}_{u,i}$) and the down-travel actual average characteristics ($\bar{y}_{d,i}$)from the reference straight line.

NOTE 1 When making mention of ξ_{LH} , indication should be made of the kind of reference straight line.

NOTE 2 When working characteristics is used as the reference characteristics, the ξ_{LH} obtained may probably include components of repeatability, and therefore is, in a strict sense, not an indication of pure combined linearity and hysteresis (ξ_{LH}).

NOTE 3 For non-linear transducers, combined conformity and hysteresis (ξ_{CH}) may be calculated in a similar way.

5.1.2 The calculation of reference line

A straight-line fitting is made of the up-travel actual average characteristics ($\bar{y}_{u,i}$) and the down-travel actual average characteristics ($\bar{y}_{d,i}$) by using a best straight line. A detailed illustration of the calculation is shown in relevant parts of Annexes A, B and C.

5.2 Combined linearity, hysteresis and repeatability (ξLHR)

This is also called the Linearity plus hysteresis plus repeatability.

In this case, the reference straight line is just the working straight line.

5.2.1 The general form of calculating formula

$$\xi_{LHR} = \pm \frac{Max\left|B_i + t\,0{,}95 s_i\right|}{Y_{FS}} \times 100\ \% \tag{27}$$

where

B_i is the extreme value of the total systematic error at the ith measuring point, which can be found by using conventional non-statistical methods;

$t\,0{,}95 s_i$ is the extreme value of the total random error at the ith measuring point. t 0,95 is the coverage factor with a confidence of 95 % in the t-distribution. s_i is the sample standard deviation at the ith measuring point.

Equation (2), can be also shown as:

$$\xi_{LHR} = \pm \frac{\left|\Delta Y_{LHR,max}\right|}{Y_{FS}} \times 100\ \% \tag{28}$$

where

$Y_{LHR,max}$ is the maximum deviation of the LHR extreme-point envelope from the working straight line.

NOTE 1 The ξ_{LHR} here corresponds to the Ur in Annex F.

NOTE 2 For non-linear transducers, combined conformity and hysteresis and repeatability ξ_{CHR} may be calculated in a similar way.

5.2.2 The alternative forms of the calculating formulas

Considering the difference of the up-travel actual average characteristics ($\bar{y}_{u,i}$) from the down-travel actual average characteristics ($\bar{y}_{d,i}$), Equation (3) may take the following simple forms:

$$\xi_{LHR} = \pm\ Max\left|\ \xi_{LHR,u,i},\ \xi_{LHR,d,i}\right| \tag{29}$$

$$\xi_{LHR,u,i} = \pm \frac{\left|\Delta Y_{LH,u,i} + t\,0{,}95 s_{u,i}\right|}{Y_{FS}} \times 100\% \tag{30}$$

$$\xi_{LHR,d,i} = \pm \frac{\left|\Delta Y_{LH,d,i} + t\,0{,}95 s_{d,i}\right|}{Y_{FS}} \times 100\% \tag{31}$$

where

$\xi_{LHR,u,i}$ is the total uncertainty at the ith measuring point in the upper-travel;

$\xi_{LHR,d,i}$ is the total uncertainty at the ith measuring point in the down-travel;

$\Delta Y_{LH,u,i}$ is the deviation of the up-travel actual average characteristics ($\bar{y}_{u,i}$) from the working characteristics at the ith measuring point;

$\Delta Y_{LH,d,I}$ is the deviation of the down-travel actual average characteristics ($\bar{y}_{d,i}$) from the working characteristics at the ith measuring point;

$s_{u,i}$ is the sample standard deviation at the ith measuring point in the up-travel;

$s_{d,I}$ is the sample standard deviation at the ith measuring point in the down-travel;

t 0,95 is the coverage factor with a confidence level of 95 % in the t-distribution.

NOTE The adoption of the method of extreme-point envelope in 5.2.3.2 to calculate ξ_{LHR} is most desirable. The method is clear in concept and simple in calculation.

5.2.3 The method for calculating the working characteristics

5.2.3.1 The general principles governing the choice and calculation of the working characteristics

The following points should be taken into consideration:

- The way in which the transducer under calibration is used.
- The simplification of transducer application.
- The possibility to have a better value of total uncertainty.

5.2.3.2 The method of L(C)HR extreme-point envelope

In general, the total uncertainty of a transducer depends on the combined contribution of linearity (conformity), hysteresis and repeatability. As shown in Figure 9, we can find, at the x_i measuring point, the up-travel average point $\overline{y}_{u,i}$ and the down-travel average point $\overline{y}_{d,i}$ respectively. According to 4.7.3, we can also find, at the x_i measuring point, the up-travel sample standard deviation $s_{u,i}$ and the down-travel sample standard deviation $s_{d,i}$ respectively. In order to find the two extreme-points at the x_i point , $cs_{u,i}$ is subtracted from $cs_{u,i}$, and $cs_{d,i}$ is added to $cs_{d,i}$. Thus at the x_i measuring point, we obtain two extreme-points, namely:

$$y_{u,i,\,min} = \overline{y}_{u,i} - cs_{u,i} \quad (32)$$

$$y_{d,i,\,max} = \overline{y}_{d,i} + cs_{d,i} \quad (33)$$

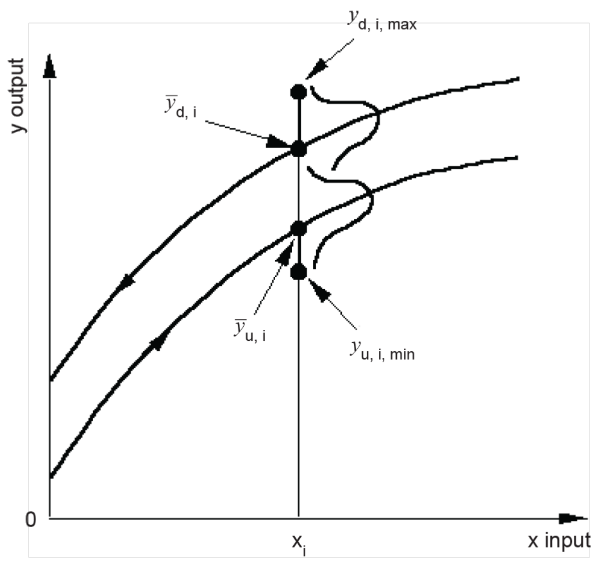


Figure 9 – The method of L(C)HR extreme-point envelope

By doing this way, we can obtain 2m (m is the number of the measuring points) extreme points, and thus forming an envelope, namely an actual uncertainty zone, in which the true values of the measurements lie at the given confidence level.

With this actual uncertainty zone, we can easily draw a best straight line to approximate the zone, thus obtaining the transducer's working straight line. After having found the working straight line and by using Equation (3), the calculation of the total uncertainty becomes quite easy. For detailed illustration, please see relevant parts in Annexes A, B and C.

NOTE This method can be regarded as a geometrical method, clear in concept and easy to use. It provides a most effective way to solve for the combined linearity (conformity) plus hysteresis plus repeatability (ξL(C) HR), and therefore the working characteristics of the transducer.

Annex A
(informative)

Methods and examples for calculating linearities

A.1 Numerical examples for calculating zero-based linearity

A group of calibration data is given in Table A.1, calculate the zero-based straight line and the zero-based linearity.

Table A.1

Input x	0,00	1,00	2,00	3,00	4,00	5,00
Output y	0,03	10,05	20,20	29,60	39,90	50,00

A.1.1 The general principle of calculation

Zero-based straight line is a line that goes through the theoretical zero point (x = 0, y = 0) and the center of gravity of the two points which respectively have minimal positive and negative deviations. As it is not possible to find the zero-based straight line directly, therefore multiple approximations are the only way to do the job.

A.1.2 Solving for the first approximating straight line

The line going through the theoretical zero point (x = 0, y = 0) and the actual rear end is chosen as the first approximating straight line, with the equation form as: $Y_{ze,(1)} = 10,00x$. The deviations of the actual average characteristics (curve) from the first approximating straight line are shown in Table A.2.

Table A.2

x	0,00	1,00	2,00	3,00	4,00	5,00
$\Delta Y_{ze,(1)}$	+0,030	+0,050	+0,200	-0,400	-0,100	0,000

A.1.3 Solving for the second approximating straight line

From Table A.2, we find that at points 3 and 4, namely at the two points with x = 2.00 and x = 3.00, there appear, respectively, the maximum positive deviation and the maximum negative deviation. The next step is to find the coordinates of the centre of gravity of the two points, namely point 3 and 4.

$$x_{3,4} = \frac{2+3}{2} = 2,50\,; \quad y_{3,4} = \frac{20,20+29,60}{2} = 24,90$$

Therefore the equation for the second approximating straight line is:

$$Y_{ze,(2)} = \frac{y_{3,4}}{x_{3,4}}x = \frac{24,90}{2,50}x = 9,9600x$$

The deviations from the second approximating straight line is:

Table A.3

x	0,00	1,00	2,00	3,00	4,00	5,00
$\Delta Y_{ze,(2)}$	+0,030	+0,090	+0,280*	-0,280*	+0,060	+0,200

Obviously from Table A.3, at points 3 and 4, there appears the maximum positive deviation and the maximum negative deviation, being minimized and equal to each other in absolute value. Therefore, the second approximating straight line is the zero-based straight line we want to obtain. The zero-based linearity is:

$$L_{ze} = \pm \frac{0{,}280}{(5\text{-}0) \times 9{,}9600} \times 100\ \% = \pm 0{,}562\ \%$$

NOTE 1 The terminal-based line and terminal-based linearity for the data in this example are:

$$Y_{te} = 0{,}0300 + 9{,}9940x;\ L_{ze} = -0{,}825\ \%;$$

NOTE 2 The front-terminal-based line and terminal-based linearity for the data in this example are:

$$Y_{f.te} = 0{,}0300 + 9{,}9480x;\ L_{f.te} = \pm 0{,}551\ \%\ 。$$

A.2 Numerical examples for calculating independent linearity

Solve for the best straight line and independent linearity for a group of calibration data. The original data and the intermediate results of calculation are given in Table A.4.

Table A.4

Input x	1,00	2,00	3,00	4,00	5,00	6,00
Output y	2,02	4,00	5,98	7,90	10,10	12,05
Deviations from the terminal-based straight line ΔYte	0,00	-0,026	-0,052	-0,138	+0,056	0,00
ΔY'te = 100 ΔYte	0,00	-2,60	-5,20	-13,80	+5,60	0,00

A.2.1 The principle of a precise method

The key to the calculation of the independent linearity is to find the best straight line. There are two ways to do the job: analytical method and graphical method, with the latter being easy and straight-forward. Our illustration is focused on graphical method.

The precise method used in this report for finding the best straight line is called the Method of Sun's convex polygon, which was first published in [08] in Annex G. The method is simple and yet powerful in working out computer programs as well as in calculating manually with its modified form, the Method of Transformed Convex Polygon. As it is almost impossible to build the original convex polygon manually, the following is a procedure to find the best straight line with the help of the transformed convex polygon.

A.2.1.1 Finding the terminal-based straight line of the calibration data

It is easy to obtain: $Y_{te} = 0{,}0140 + 2{,}0060x$

The deviations of the calibration data from the terminal-based straight line are shown in Table A.4.

A.2.1.2 Amplifying the deviations

For easy graph making, the deviations are amplified by the same factor. In our case the factor is 100 and the amplified deviations are also shown in Table A.4

A.2.1.3 Finding the transformed convex polygon

Mark all the deviation points on a coordinate graph paper and draw a convex polygon from the points, with all the points either inside the polygon or at its sides as shown in Figure A.1.

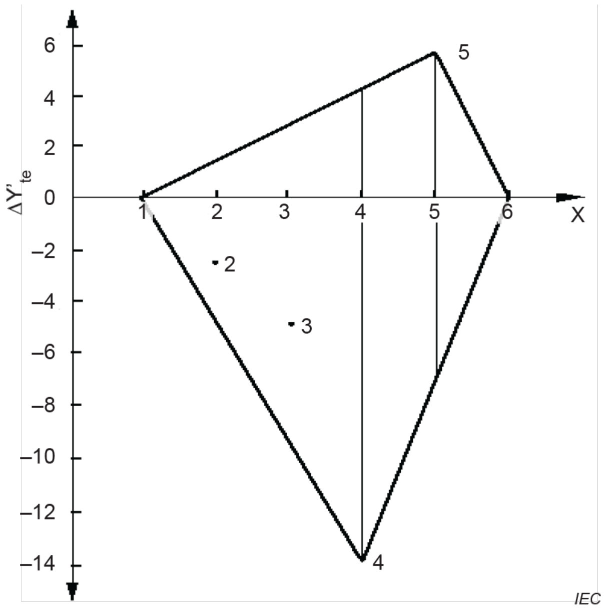


Figure A.1 – The transformed convex polygon

A.2.1.4 Drawing the vertical lines and finding the longest one

Draw vertical lines from all the vertices of the polygon to their opposite sides or bases. Find the longest vertical line within the polygon. From our visual observation, the vertical line drawn from vertex 4 is the longest.

If it is difficult to find the longest vertical line within the transformed convex polygon by visual observation, then by using the following formulas, the lengths of all the vertical lines within the original convex polygon can be calculated as follows. Suppose we want to find the length (absolute value) of the vertical line which is drawn from vertex 4 to side 1 and 5:

$$\Delta y_{(1,\,4,\,5)} = \frac{\left| y_1(x_4 - x_5) + y_4(x_5 - x_1) + y_5(x_1 - x_4) \right|}{x_5 - x_1}$$

$$\Delta y_{(1,\,4,\,5)} = \frac{\left| 2{,}02(4{,}00 - 5{,}00) + 7{,}90(5{,}00 - 1{,}00) + 10{,}10(1{,}00 - 4{,}00) \right|}{5{,}00 - 1{,}00} = 0{,}180$$

Similarly, the length of another vertical line is:

$$\Delta y_{(4,\,5,\,6)} = \frac{\left| y_4(x_5 - x_6) + y_5(x_6 - x_4) + y_6(x_4 - x_5) \right|}{x_6 - x_4}$$

$$\Delta y_{(4,\,5,\,6)} = \frac{\left|7{,}90\ (5{,}00 - 6{,}00) + 10{,}10\ (6{,}00 - 4{,}00) + 12{,}05\ (4{,}00 - 5{,}00)\right|}{6{,}00 - 4{,}00} = 0{,}125$$

A.2.1.5 Finding the best triangle

From Figure A.1 and from our calculations above, the vertex from which the longest vertical line is drawn is point 4, and the two end points of its opposite side or base are point 1 and point 5. The three data points can form a triangle, with two sides 4-1 and 4-5 and a base 1-5. This triangle is constructed from the original calibration data, and is called the **Best Triangle** hereafter.

A.2.1.6 Drawing the best straight line

The line connecting the two midpoints of the two sides of the best triangle is just the best straight line we want. The derivation of the equation for the best straight line is as follows. First calculate the coordinates of the midpoint of side 4-1:

$$x_{1,4} = \frac{x_1 + x_4}{2} = \frac{1{,}00 + 4{,}00}{2} = 2{,}50\,;\ y_{1,4} = \frac{y_1 + y_4}{2} = \frac{2{,}02 + 7{,}90}{2} = 4{,}96$$

Then calculate the coordinates of the midpoint of side 4-5:

$$x_{4,5} = \frac{x_4 + x_5}{2} = \frac{4{,}00 + 5{,}00}{2} = 4{,}50\,;\ y_{4,5} = \frac{y_4 + y_5}{2} = \frac{7{,}90 + 10{,}10}{2} = 9{,}00$$

Therefore the slope of the best straight line is:

$$b = \frac{y_{4,5} - y_{1,4}}{x_{4,5} - x_{1,4}} = \frac{9{,}00 - 4{,}96}{4{,}50 - 2{,}50} = 2{,}0200$$

The intercept of the best straight line is:

$$a = y_{1,4} - bx_{1,4} = 4{,}96 - 2{,}02 \times 2{,}5 = -0{,}0900$$

Finally, the equation for the best straight line is:

$$Yin = -0{,}0900 + 2{,}0200x$$

The deviations of the actual average curve from the best straight line are:

Table A.5

X	1,00	2,00	3,00	4,00	5,00	6,00
Δy_{in}	+0,090 *	+0,050	+0,010	-0,090 *	+0,090 *	+0,020

From Table A.5, we can see that we have three points (with asterisk) where the signs of the same maximum deviation appear in an sign-alternating way:

$$\Delta Y_{in,max} = \pm 0{,}090$$

Actually, the absolute value of the maximum deviation is just half of the length of the longest vertical line. It is the smallest maximum deviation that can be obtained. The principle of alternating points, where the same smallest maximum deviations appear with alternating signs, is a powerful criterion for the existence of a best straight line or a best curve. No other fitting straight lines or curves have this peculiarity.

A.2.1.7 Calculating the independent linearity

$$L_{in} = \pm \frac{\Delta Y_{in,max}}{\left(Y_{in,max} - Y_{in,min}\right)} = \pm \frac{\Delta Y_{in,max}}{b\left(x_{max} - x_{min}\right)}$$

$$= \pm \frac{0{,}090}{2{,}0200\left(6{,}00 - 1{,}00\right)} = \pm 0{,}891\ \%$$

A.2.2 The principle of the makeshift methods

The shifted-terminal-based straight line and the shifted-least-squares line can be used as a makeshift for best straight line.

A.2.2.1 The shifted-terminal-based straight line

There are several ways to find the equation of the shifted-terminal-based straight line. Here, we just show the equation and the corresponding linearity:

$$Y_{s.te} = -0{,}0270 + 2{,}0060x \quad L_{s.te} = \pm 0{,}97\ \%$$

A.2.2.2 The least-squares straight line

The equation for the least-squares straight line and the corresponding linearity:

$$Y_{ls} = -0{,}0287 + 2{,}0106x \quad L_{ls} = -1{,}13\ \%$$

A.3 A comparison of the results of all kinds of linearities

For the same calibration data in Table A.4, the results of all kinds of linearities are listed below for comparison. Here obviously, the independent linearity is the highest linearity. It is recommended that independent linearity is used whenever there is a requirement for strict evaluation of linearity.

Independent linearity	L = ±0,89 %
Shifted-least-squares linearity	L = ±0,95 %
Shifted-terminal-based linearity	L = ±0,97 %
Zero-based linearity	L = ±1,00 %
Front-terminal-based linearity	L = ±1,03 %
Least-squares linearity	L = −1,13 %
Terminal-based linearity	L = −1,38 %

Annex B
(informative)

Methods and Examples for Calculating Conformities

B.1 The general principle for calculating conformities

The method for finding the reference curves for calculating conformities is based on Chebyshev's theory of best approximation. More specifically, it is based on the principle of alternating points and an extended Remez's algorithm. Here, the fitting curves exclusively take the form of an algebraic polynomial of a certain degree. The main points of the algorithm are as follows.

B.1.1 Determining the degree of the fitting curves

The following hints may be helpful:

1) According to the fitting accuracy needed to meet the conformity requirement;
2) According to the general shape of the actual average curve and by experience;
3) From an equally-spaced difference quotient table;
4) By a specially-created computer program.

B.1.2 Choosing the number of the alternating points

1) If there are n unknowns in the polynomial, (n+1) = k alternating points should be chosen;
2) If the polynomial is required to pass through p points, the number of the alternating points should be r
3) Distributing n points on x-axis in an orderly way. In the first approximating calculation, the n points may be placed roughly equally spaced, and preferably, with the front and the rear ends of the actual average curve included. Find the first approximating curve which goes through the n points. Find the deviations of the actual average curve from the first approximating curve.

B.1.3 Determining the locations of the alternating points

Starting from the second approximation, the determination of the locations of the k alternating points will proceed according to the following considerations:

1) The deviations of the points chosen should be sign-alternating;
2) The points where there exists the largest positive deviation or the largest negative deviation should be first included in the candidate alternating points. The larger the deviation of a point is, the more possible the point is to be chosen;
3) Zero deviation may be regarded as the smallest positive deviation or as the smallest negative deviation.

B.1.4 Finding the finally-successful alternating points

The whole process in which the finally-successful alternating points are found is an iterative one. Once we find the deviations at the alternating points to be sign-alternating, equal in absolute value, and the largest of all. This is an indication that we have arrived at the finally-successful alternating points. The fitting curve corresponding to the finally-successful alternating points is just the reference curve needed. The deviations at the alternating points are the smallest for the same group of calibration data, for the same form and degree of the fitting curve, and for the same constraints put upon.

B.2 Numerical examples for calculating conformities

A group of calibration data of a non-linear transducer is given in Table B.1. Solve for the terminal-based curve and terminal-based conformity, the zero-based curve and zero-based conformity, the front-terminal-based curve and front-terminal-based conformity, the best curve and independent conformity, the least-squares curve and the least-squares conformity. At last, give a short discussion of the way the theoretical curve is chosen.

Table B.1

Input x	0,00	1,00	2,00	3,00	4,00	5,00
Output y	0,10	1,00	1,80	2,60	3,00	3,80

Solution: Looking at Figure B.1, by our experience, the form of the fitting curve is chosen as a quadratic polynomial $Y = a+bx+cx^2$.

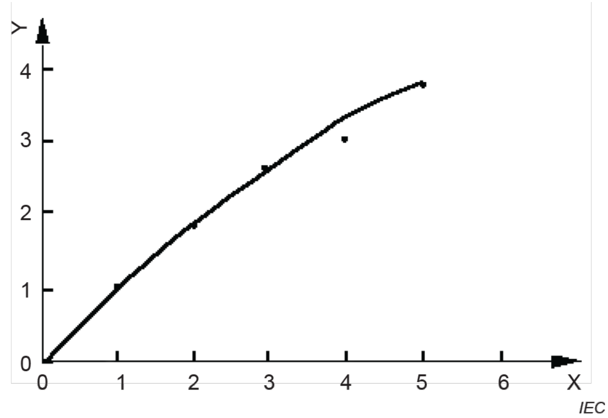


Figure B.1 – The curve roughly drawn from the given data

B.2.1 Solving for the terminal-based curve of the second degree and the terminal-based conformity of the second degree

B.2.1.1 Finding the first approximating curve

We know that three un-collinear points on a plane will determine exclusively a quadratic curve in the form of a algebraic polynomial. Therefore, we can choose 3 points from Table B.1, roughly equally-spaced and with the first and last points included:

$$x = 0,00,\ y = 0,10;\ x = 3,00,\ y = 2,60;\ x = 5,00,\ y = 3,80$$

Create a linear system of equations with 3 unknowns:

$$0,10-[a+b(0,00)+c(0,00)^2] = 0,00$$

$$2,60-[a+b(3,00)+c(3,00)^2] = 0,00$$

$$3,80-[a+b(5,00)+c(5,00)^2] = 0,00$$

Solve for the 3 coefficients, and we have:

$a = 0,1000$; $b = 0,9733$; $c = -0,0467$

Hence the equation of the first approximating curve is:

$Y_{te,(1)} = 0,1000 + 0.9733x - 0,0467x^2$

The deviations at the calibration points are:

Table B.2

X	0,00	1,00	2,00	3,00	4,00	5,00
$\Delta Y_{te,\ (1)}$	0,000	-0,027	-0,060	+0,000 *	-0,246 *	0,000

B.2.1.2 Finding the second approximating curve

According to the explanations in B.2.1.1, we know that a quadratic polynomial has 3 unknowns, so we have to choose 3 + 1 = 4 alternating points. Since in this case what we need is a terminal-based curve which should pass through 2 fixed points, so the actual number of alternating points is 4 – 2 = 2. From Table B.2, we find at x = 3,00, there is the largest positive deviation +0,000, and at x = 4,00, there is the largest negative deviation –0,246, as shown by the asterisks. The 2 points are chosen as the alternating points:

$$x = 3,00,\ y = 2,60;\ x = 4,00,\ y = 3,00$$

As we hope that at alternating points, the deviations should be sign-alternating and of the same largest absolute value, therefore we can construct a complementary equation:

$$2,60 - [a + b(3,00) + c(3,00)^2] = -\{3,00 - [a + b(4,00) + c(4,00)^2]\}$$

This equation and the other 2 equations form the simultaneous equations in 3 variables:

$$0,10 - [a + b(0,00) + c(0,00)^2] = 0,00$$

$$3,80 - [a + b(5,00) + c(5,00)^2] = 0,00$$

$$(2,60 + 3,00) - [2a + b(3,00 + 4,00) + c(3,00^2 + 4,00^2)] = 0,00$$

Solve the equations for the unknowns, we have:

$$a = 0,1000;\ b = 0,8500;\ c = -0,0220$$

Hence the equation of the second approximating curve of the terminal-based curve of the second degree is:

$$Y_{te,\ (2)} = 0,1000 + 0,8500x - 0,0220x^2$$

The deviations at the calibration points are:

Table B.3

X	0,00	1,00	2,00	3,00	4,00	5,00
$\Delta Y_{te,\ (2)}$	0,000	+0,072	+0,088	+0,148 *	-0,148 *	0,000

From Table B.3, we can see that at the 2 alternating points the deviations are sign-alternating and of the same largest absolute value ±0,148, as shown by the asterisks. Therefore we have arrived at the finally-successful alternating points and the second approximating curve is the terminal-based curve of the second degree.

B.2.1.3 Finding the terminal-based conformity of the second degree

$$C_{te} = \pm \frac{\Delta Y_{te,max}}{Y_{te,max} - Y_{te,min}} = \pm \frac{0,148}{3,80 - 0,10} = \pm 4,000\ \%$$

B.2.2 Solving for the zero-based curve of the second degree and the zero-based conformity of the second degree

In this case, the fitting curve is forced to pass the theoretical zero (x = 0, y = 0), so the coefficient of the constant term of the quadratic polynomial a = 0, and the fitting curve takes the form $Y = bx + cx^2$. As we have only 2 unknowns, the number of the alternating points is 2 + 1 = 3. Suppose the absolute value of the deviations at the alternating points is μ.

B.2.2.1 Finding the first approximating curve

From Table B.3, and taking into consideration of the fact that zero deviation may be regarded as the smallest positive deviation as well as the smallest negative deviation, we can choose the following 3 points as the alternating points:

x = 3,00, y = 2,60; x = 4,00, y = 3,00; x = 5,00, y = 3,80

From the 3 points we can construct the simultaneous equations in 3 variables:

$2,60 - [b(3,00) + c(3,00)2] = \mu$

$3,00 - [b(4,00) + c(4,00)2] = -\mu$

$3,80 - [b(5,00) + c(5,00)2] = \mu$

Solve the equations for the unknowns, we have:

b = 0,9613; c = −0,0452

Hence the equation of the first approximating curve of the zero-based curve of the second degree is:

$Y_{ze,(1)} = 0.9613x - 0.0452x^2$

The deviations at the calibration points are:

Table B.4

X	0,00	1,00	2,00	3,00	4,00	5,00
$\Delta Y_{ze,\ (1)}$	+0,100	+0,084	+0,058	+0,123 *	-0,122 *	+0,124 *

From Table B.4, we can see that at the 3 alternating points the deviations are sign-alternating and of the same largest absolute value ±0.123, namely μ = 0.123, as shown by the asterisks. Therefore we have arrived at the finally-successful alternating points and this first approximating curve is the zero-based curve of the second degree we need. The slight difference of the 3 deviations from one another is caused by the limited number of significant digits used in computing.

B.2.2.2 Finding the zero-based conformity of the second degree:

$$C_{ze} = \pm \frac{\Delta Y_{ze,max}}{Y_{ze,max} - Y_{ze,min}} = \pm \frac{0,123}{3,677 - 0,00} = \pm 3,345\ \%$$

Obviously, it is a little better than the terminal-based conformity of the second degree.

B.2.3 Solving for the front-terminal-based curve of the second degree and the front-terminal-based conformity of the second degree

In this case the fitting curve is forced to pass the front end (x = 0, y = 0,1), so the coefficient of the constant term of the quadratic polynomial a = 0,1, and the fitting curve takes the form Y = 0,1 + bx + cx2. As we have only 2 unknowns, the number of the alternating points is 2 + 1 = 3. Suppose the absolute value of the deviations at the alternating points is μ.

B.2.3.1 Finding the first approximating curve

Here we choose the same 3 alternating points as those we choose for the first approximating curve in solving the zero-based curve of the second degree. Thus we can construct the simultaneous equations in 3 variables:

$$2,60 - [0,10 + b(3,00) + c(3,00)^2] = +\mu$$

$$3,00 - [0,10 + b(4,00) + c(4,00)^2] = -\mu$$

$$3,80 - [0,10 + b(5,00) + c(5,00)^2] = +\mu$$

Solve the equations for the unknowns, we have:

$$a = 0,1000;\ b = 0,9097;\ c = -0,0387$$

$$Y_{f.te,\ (1)} = 0,1000 + 0,9097x - 0,0387x^2$$

The deviations at the calibration points are:

Table B.5

X	0,00	1,00	2,00	3,00	4,00	5,00
$\Delta Y_{f.te,\ (1)}$	0,000	+0,029	+0,035	+0,119 *	-0,119 *	+0,119 *

From Table B.5, we can see that at the 3 alternating points the deviations are sign-alternating and of the same largest absolute value ±0,119, namely μ = 0,119, as shown by the asterisks. Therefore we have arrived at the finally-successful alternating points and this first approximating curve is the front-terminal-based curve of the second degree we need.

B.2.3.2 Finding the front-terminal-based conformity of the second degree

$$C_{f.te} = \pm \frac{\Delta Y_{f.te,max}}{Y_{f.te,max} - Y_{f.te,min}} = \pm \frac{0{,}119}{3{,}681 - 0{,}10} = \pm 3{,}323\ \%$$

It is a little better than the zero-based conformity of the second degree. We can also see that at the front end the deviation is zero and remains rather small until x = 2. If very small deviations are desired around the front end, in this case the front-terminal-based curve of the second degree may be a better choice.

B.2.4 Solving for the best curve of the second degree and the independent conformity of the second degree

In this case there is no constraint of any kinds applied on the quadratic polynomial, so we have 4 alternating points. Usually, roughly equally-spaced 3 points, preferably including the front end and the rear end, may be chosen to construct the first approximating curve. Then find the finally-successful alternating points step by step.

As we already have the front-terminal-based curve, by studying Table B.5, it is very easy to find 4 points as the candidate alternating points and construct the simultaneous equations in 4 variables as our first approximating curve. As we did before, the absolute value of the deviations at the alternating points is taken as μ.

B.2.4.1 Finding the first approximating curve

The 4 candidate alternating points are:

x = 0,00, y = 0,10; x = 3,00, y = 2,60;

x = 4,00, y = 3,00; x = 5,00, y = 3,80

The simultaneous equations in 4 variables:

$$0{,}10 - [a + b(0{,}00) + c(0{,}00)^2] = +\mu$$

$$2{,}60 - [a + b(3{,}00) + c(3{,}00)^2] = -\mu$$

$$3{,}00 - [a + b(4{,}00) + c(4{,}00)^2] = +\mu$$

$$3{,}80 - [a + b(5{,}00) + c(5{,}00)^2] = -\mu$$

Solve the equations for the unknowns, we have:

a = 0,2156; b = 0,8500; c = −0,0312

Hence the equation of the first approximating curve of the best curve of the second degree is:

$$Y_{in,\,(1)} = 0{,}2156 + 0{,}8500x - 0{,}0312x^2$$

The deviations at the calibration points are listed in Table B.6.

Table B.6

x	0,00	1,00	2,00	3,00	4,00	5,00
$\Delta Y_{in,\,(1)}$	-0,116*	-0,034	+0,009	+0,116 *	-0,116 *	+0,116 *

From Table B.6, we can see that at the 4 alternating points the deviations are sign-alternating and of the same largest absolute value ±0,116, namely μ = 0,116, as shown by the asterisks. Therefore we have arrived at the finally-successful alternating points and this first approximating curve is the best curve of the second degree we need.

B.2.4.2 Finding the best conformity of the second degree

$$C_{in} = \pm \frac{\Delta Y_{in,max}}{Y_{in,max} - Y_{in,min}} = \pm \frac{0{,}116}{3{,}684 - 0{,}216} = \pm 3{,}345\ \%$$

NOTE In terms of the numerical value, the best conformity of the second degree is slightly larger than the front-terminal-based conformity of the second degree. This is caused by a smaller Y_{FS} for the former. The maximum deviation ΔY = ±0,116 is the smallest for all kinds of fitting curves in this example. If the number of significant digits is large enough in computing, we will arrive at the result that the best conformity of the second degree and the front-terminal-based conformity of the second degree, in this case, are the same, namely ±3,333 %

B.2.5 Solving for the least-squares curve of the second degree and the least-squares conformity of the second degree

The least-squares curve of the second degree takes the following equation form:

$$Y_{ls} = a + bx + cx^2$$

The formulas for calculating the coefficients, which are useful in creating computer programs, are as follows:

$$c = \frac{\Sigma(x^2, y) \cdot \Sigma(x, x) - \Sigma(x,y) \cdot \Sigma(x, x^2)}{\Sigma(x\,,x) \cdot \Sigma\left(x^2, x^2\right) - \left\{\Sigma\left(x\,, x^2\right)\right\}^2}$$

$$b = \frac{\Sigma(x, y) \cdot \Sigma(x^2, x^2) - \Sigma(x^2, y) \cdot \Sigma(x, x^2)}{\Sigma(x\,,x) \cdot \Sigma\left(x^2, x^2\right) - \left\{\Sigma\left(x\,, x^2\right)\right\}^2}$$

$$a = \frac{\Sigma y}{m} - b\frac{\Sigma x}{m} - c\frac{\Sigma x^2}{m}$$

In the above formulas, y are all actual average characteristics $\bar{y}_i$ and the notation ∑(j ,k), which appears several times, should be recovered to its real form in actual calculation:

$$\sum(j\,,k) = \sum(j \cdot k) - (\sum j \cdot \sum k)/m$$

In this example, i = 1~m , and m = 6. So the least-squares curve of the second degree is:

$$Y_{ls} = 0{,}1179 + 0{,}9104x - 0{,}0375x^2$$

The deviations at the calibration points are shown in Table B.7.

Table B.7

X	0,00	1,00	2,00	3,00	4,00	5,00
ΔY_{ls}	-0,018	+0,009	+0,011	+0,089	-0,159 *	+0,068

The least-squares conformity of the second degree:

$$C_{is} = \frac{\Delta Y_{is,max}}{Y_{is,max} - Y_{is,min}} = \frac{0{,}159}{3{,}7324 - 0{,}1179} = -4{,}399\%$$

The conformity is much (about 32 %) larger than the best conformity of the second degree. Its shortage is self-evident.

B.2.6 A rough principle guiding the choice of the theoretical curve

1) According to the requirements of the system in which the transducer is only a component. In this case the theoretical curve is mostly pre-specified. But this may result in more difficulties in transducer manufacturing.
2) According to the working curve obtained in the calibration test, as this may make full use of the performance of the already-existing transducer.

Annex C
(informative)

Examples for calculating transducer individual and combined performance indicators

C.1 General principles of calculation

1) Make an examination to see if there are any suspect data points and unreasonable data points (usually caused by temperature contributions) in the original calibration data. If there are any such data points, effort should be made to identify and locate the trouble sources and get rid of them. Then do the calibration again. Only the data free from any suspect and unreasonable data points are suitable for calculating the performance indicators.
2) Make use of the method of L(C)HR extreme-point envelope to calculate the working characteristics and the total uncertainty. The calculation of individual performance indicators remains to be done by conventional methods.

NOTE It is suggested that the statistical detection method and the panorama-visual inspection method in Annex E may be used for discovering the suspect data points, while the method of comparison of adjacent-in-cycle data pair and the panorama-visual inspection method in Annex E are used for discovering the unreasonable data points. It is not advisable to discard the suspect or unreasonable data points by statistical methods. The correct altitude toward dealing with them is: neither to accept nor to discard them rashly. Instead, try hard to discover the real reasons for their existence, to remove the troubles and to start a new calibration test again for trouble-free original data.

C.2 Numerical examples

C.2.1 Numerical example 1

Table C.1 shows the original calibration data of a certain linear transducer. The data, according to our inspection, are free from suspect and unreasonable data points. Calculate the individual and combined performance indicators of the transducer.

Table C.1 – The original data obtained in the calibration

Travel	Input (x)	Transducer output (y)				
		y_1	Y_2	Y_3	y_4	y_5
↓ up	0,0	0,66	0,65	0,78	0,67	0,80
	2,0	190,9	191,1	190,3	190,8	190,4
	4,0	382,8	382,3	383,5	381,8	382,8
	6,0	574,5	576,4	576,0	576,2	575,4
	8,0	769,4	769,2	770,4	769,8	771,5
	10,0	963,9	963,1	965,2	964,7	966,0
Down ↑	10,0	964,2	963,1	966,5	965,7	967,2
	8,0	770,6	772,4	771,0	770,8	772,1
	6,0	577,9	577,4	577,1	578,1	578,3
	4,0	384,0	384,8	384,2	384,9	384,2
	2,0	191,6	192,2	191,8	191,5	191,9
	0,0	1,66	1,65	1,54	1,47	1,66

C.2.1.1 Listing the intermediate calculation results

All the intermediate calculation results are listed in Table C.2. According to the methods in Clause A.2, we obtain the best straight line of the actual average characteristics:

$$y_L = -0{,}4592 + 96{,}4006x$$

and the best straight line of the up-travel actual average characteristics and the down-travel actual average characteristics:

$$y_{LH} = -0{,}7108 + 96{,}4144x$$

Table C.2 – The intermediate results of calculation

Transducer Input (x)	0,0	2,0	4,0	6,0	8,0	10,0	Remark
Up-travel average output ($\bar{y}_{u,i}$)	0,712	190,70	382,64	575,70	770,06	964,58	
Down-travel average output ($\bar{y}_{d,i}$)	1,596	191,80	384,42	577,76	771,38	965,74	
Hysteresis ($\triangle Y_H$)	0,884	1,100	1,780	2,060	1,320	1,160	$\triangle y_{H,max}$ = 2,060
The best fitting line of the up-travel and down-travel average characteristics(Y_{LH})	-0,711	192,12	384,95	577,78	770,60	963,43	Y_{FS} = 964,14
The up-travel linearity and hysteresis deviations ($\triangle y_{u,LH}$)	1,423	-1,418	-2,307 *	-2,076	-0,544	1,146	$\triangle Y_{LH,max}$ = ±2,307 Calculated from the best fitting line of the up-travel and down-travel average characteristics(y_{LH})
The down-travel linearity and hysteresis deviations ($\triangle y_{d,LH}$)	2,307 *	-0,318	-0,527	-0,016	0,776	2,307 *	
The actual average characteristics ($\bar{y}_i$)	1,154	191,24	383,53	576,73	770,72	965,16	
The best fitting line of the actual average characteristics (Y_L)	-0,459	192,34	385,14	577,94	770,75	963,55	Y_{FS} = 964,00
Linearity deviations ($\triangle Y_L$)	1,613 *	-1,092	-1,613 *	-1,214	-0,026	1,613 *	$\triangle Y_{L,max}$ = ±1,613 Calculated from the best fitting line of the actual average characteristics (Y_L)
The up-travel standard deviations (s_u)	0,072	0,339	0,635	0,768	0,926	1,130	$S_{u,max}$ = 1,130
The down-travel standard deviations (s_d)	0,087	0,274	0,402	0,498	0,814	1,172	$s_{d,max}$ = 1,172

C.2.1.2 Finding the extreme points

In order to build the extreme-point envelope, we have first to find the 2m = 12 extreme points according to the method in 5.2.3.2. All the intermediate calculation results are listed in Table C.3.

Table C.3 – Finding the extreme points n = 5 c = t 0.95 = 2.776

Up-Travel	Input (x)	0,0	2,0	4,0	6,0	8,0	10,0
	Average points ($\bar{y}_{u,i}$)	0,712	190,70	382,64	575,70	770,06	964,58
	$Cs_{u,i}$	0,200	0,941	1,762	2,132	2,571	3,125
	Extreme points $y_{u,I}$ = ($\bar{y}_{u,i}$ -$cs_{u,i}$)	0,512	189,76	380,88	573,57	767,49	961,46
Down-Travel	Input (x)	0,0	2,0	4,0	6,0	8,0	10,0
	Average points ($\bar{y}_{d,i}$)	1,596	191,80	384,42	577,76	771,38	965,74
	$Cs_{d,i}$	0,241	0,760	1,117	1,382	2,259	3,253
	Extreme points $y_{d,i}$ = ($\bar{y}_{d,i}$ +$cs_{d,i}$)	1,837	192,56	385,54	579,14	773,64	968,99

C.2.1.3 Finding the best fitting straight line

According to Clause A.2, the best fitting straight line (here the best working line) of the 2m = 12 extreme points:

$$Y_{in} = Y_{LHR} = -2,4445+96,7156x$$

The deviations of all the 12 extreme points from the best working line are shown in Table C.4. In Table C.4 there are also deviations of the up-travel actual average characteristics and of the down-travel actual average characteristics from the best working line.

Table C.4 – The deviations from the best working line

Input (x)	0,0	2,0	4,0	6,0	8,0	10,0	Remark
Best working line (Y_{LHR})	-2,445	190,99	384,42	577,85	771,28	964,71	Y_{FS} = 967,16
Deviations of the up-travel extreme points	2,957	-1,228	-3,540	-4,281 *	-3,792	-3,256	$\triangle Y_{LHR,max}$ = ±4,281
Deviations of the down-travel extreme points	4,281 *	1,574	1,119	1,293	2,358	4,281 *	
Deviations of the up-travel actual average characteristics	3,157 *	-0,290	-1,780	-2,150	-1,220	-0,130	$\triangle Y_{LH,max}$ = 4,041
Deviations of the down-travel actual average characteristics	4,041 *	0,810	0,000	-0,090	0,100	1,030	
Deviations of the Actual average Characteristics	3,599 *	0,260	-0,890	-1,120	-0,560	0,450	$\triangle Y_{L,max}$ = 3,599

According to the method of extreme-point envelope, the total uncertainty is numerically determined by the maximum deviation of the extreme points from the best working line. In Table C.4, we find that the 3 maximums equal-in-absolute-value and sign-alternating deviations ±4,281 (as shown by the asterisks) indicate that the 3 corresponding data points are the finally-successful alternating points. Therefore the total uncertainty of the transducer is:

$$Ur = \xi_{LHR} \pm \frac{\Delta Y_{LHR,max}}{Y_{FS}} \times 100\%$$

$$= \pm \frac{4,281}{96,716 \times (10,0 - 0,0)} \times 100\% = \pm 0,443\ \%$$

C.2.1.4 A complete list of the individual and combined performance indicators of the transducer

C.2.1.4.1 Equation of the best working straight line

The equation is calculated from the L(C)HR extreme-point envelope (actual uncertainty zone):

$$Y_{LHR} = -2,4445 + 96,7156x$$

C.2.1.4.2 Equation of the best utilization straight line

Calculated from the best working straight line:

$$X_{LHR} = 2,5275 \times 10^{-02} + 1,0340 \times 10^{-02}\ y$$

C.2.1.4.3 Linearity

Calculated from the data in Table C.2 and Table C.4 in Annex C:

$$\xi_L = \pm \frac{|\Delta Y_{L,max}|}{Y_{FS}} = \frac{1,613}{964,01} = \pm 0,167\%$$

(This is the independent linearity, calculated relative to the best reference straight line)

$$\xi_L = \pm \frac{|\Delta Y_{L,max}|}{Y_{FS}} = \frac{3,599}{967,16} = +0,372\%$$

(This is the theoretical linearity, calculated relative to the best working straight line)

C.2.1.4.4 Hysteresis

Calculated from the data in Table C.2 in Annex C:

$$\xi_H = \pm \frac{|\Delta Y_{H,max}|}{Y_{FS}} = \frac{2,060}{964,01} = +0,214\%$$

C.2.1.4.5 Linearity plus hysteresis

Calculated from the data in Table C.2 and Table C.4 in Annex C:

$$\xi_{LH} = \pm \frac{\left|\Delta Y_{LH,max}\right|}{Y_{FS}} = \frac{2,307}{964,14} = +0,239\%$$

(This is calculated relative to the best reference straight line)

$$\xi_{LH} = \pm \frac{\left|\Delta Y_{LH,max}\right|}{Y_{FS}} = \frac{4,041}{967,16} = +0,418\%$$

(This is calculated relative to the best working straight line)

C.2.1.4.6 Repeatability

Calculated from the data in Table C.2 in Annex C:

$$\xi_{R} = \pm \frac{\left|cs_{max}\right|}{Y_{FS}} = \frac{2,776 \times 1,172}{964,01} = +0,337\%$$

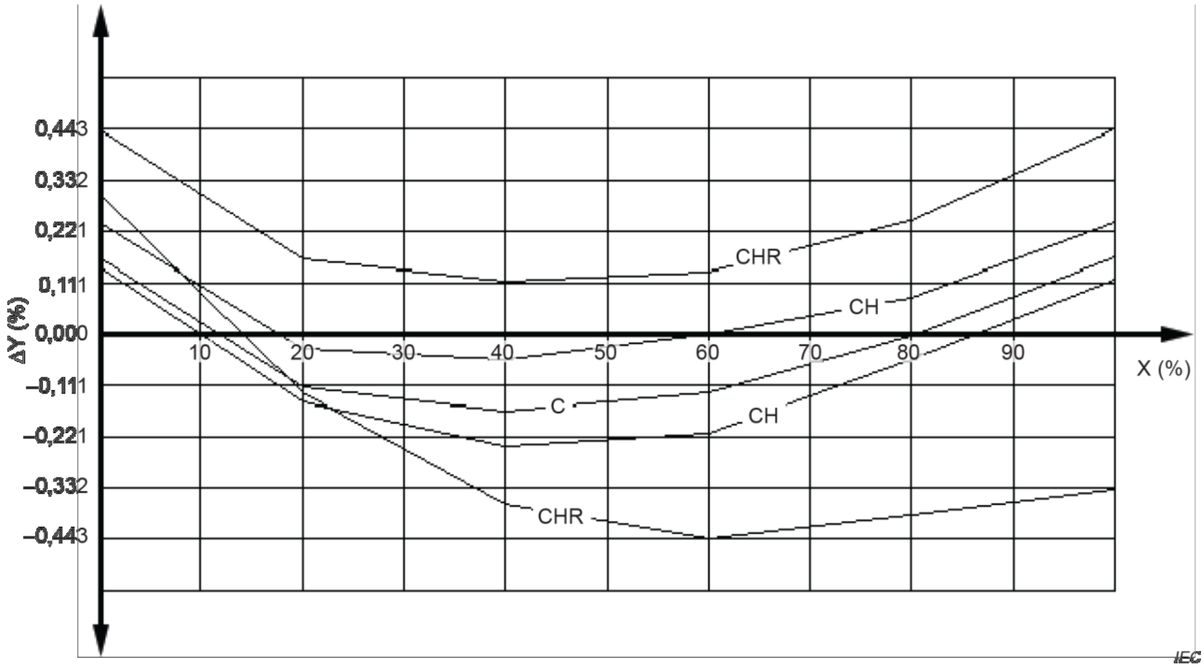
C.2.1.4.7 Total uncertainty (linearity plus hysteresis plus repeatability)

Calculated from the data in Table C.2 and Table C.4 in Annex C and with the best working straight line and the actual uncertainty zone:

$$\xi_{LHR} = \pm \frac{\left|Y_{LHR,max}\right|}{Y_{FS}} = \frac{4,281}{967,16} = \pm 0,443\%$$

NOTE In this example some performance indicators are given two results, one being relative to the best reference straight line, the other relative to the best working straight line. But it does not mean that we need to give the two results at all times. The choice is up to the requirement of the readers. If there is no special requirement, when dealing with linearity, and linearity plus hysteresis, it is advisable that our readers use only the results given relative to the best reference straight line. Without prior statement, the two performance indicators (linearity, and linearity plus hysteresis) are exclusively calculated in relation to the best reference straight line. This note also holds good for all the following examples.

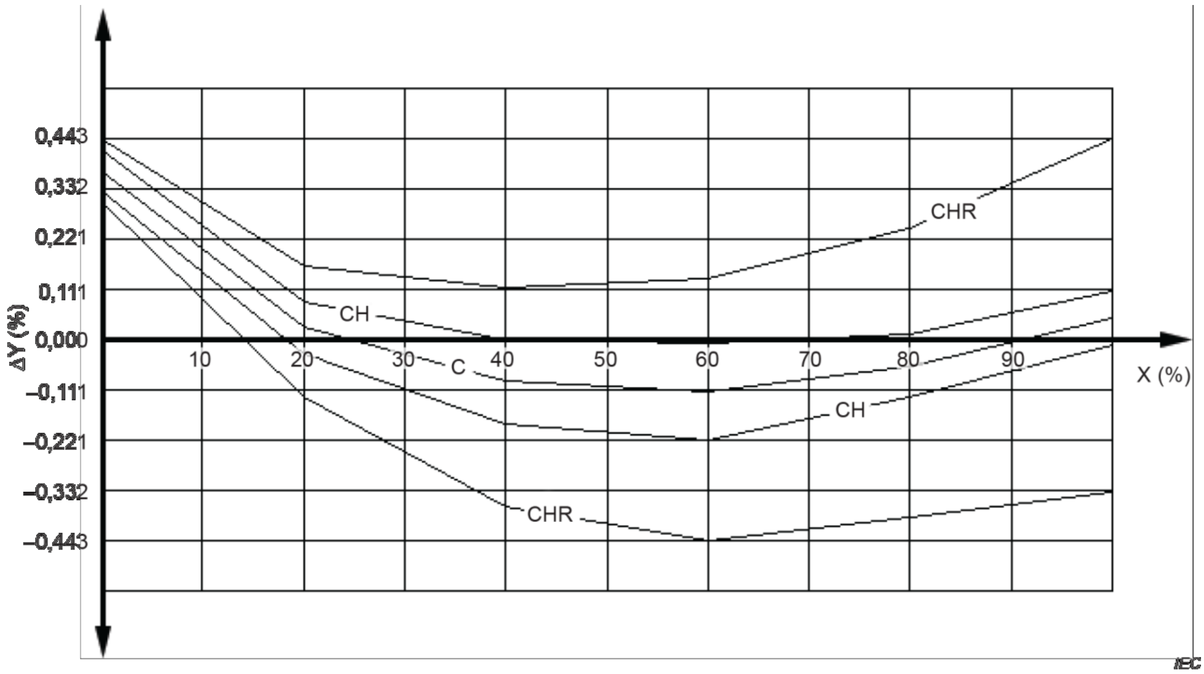
The deviation curves in this example are shown in Figure C.1 and Figure C.2.



C —– Conformity; CH —– Conformity plus Hysteresis

CHR —– Conformity plus Hysteresis plus Repeatability

Figure C.1 – Deviation curves which are calculated relative to relevant best reference lines of the first degree



C —– Conformity; CH —– Conformity plus Hysteresis

CHR —– Conformity plus Hysteresis plus Repeatability

Figure C.2 – Deviation curves which are calculated relative to the working line of the first degree

C.2.2 Numerical example 2

The original data is the same as that in numerical example 1. The only difference is that the transducer is regarded as a non-linear one and a quadratic polynomial is used to fit the data (namely the 2m = 12 extreme points). Calculate the individual and combined performance indicators of the transducer.

The calculation results are listed below, but the calculation process is omitted for brevity. The deviation curves in this example are shown in Figure C.3 and Figure C.4.

The best working curve of the second degree:

$$Y_{CHR} = -1.9318 + 96.2884x + 0.0427x^2$$

Conformities:

ξ_C = ±0,035 % (calculated relative to the best reference curve)

ξ_C = 0,319 % (calculated relative to the best working curve)

Hysteresis:

ξ_H = 0,214 %

Repeatability:

ξ_R = 0,337 % (coverage factor c = t 0,95 = 2,776)

Conformity plus hysteresis:

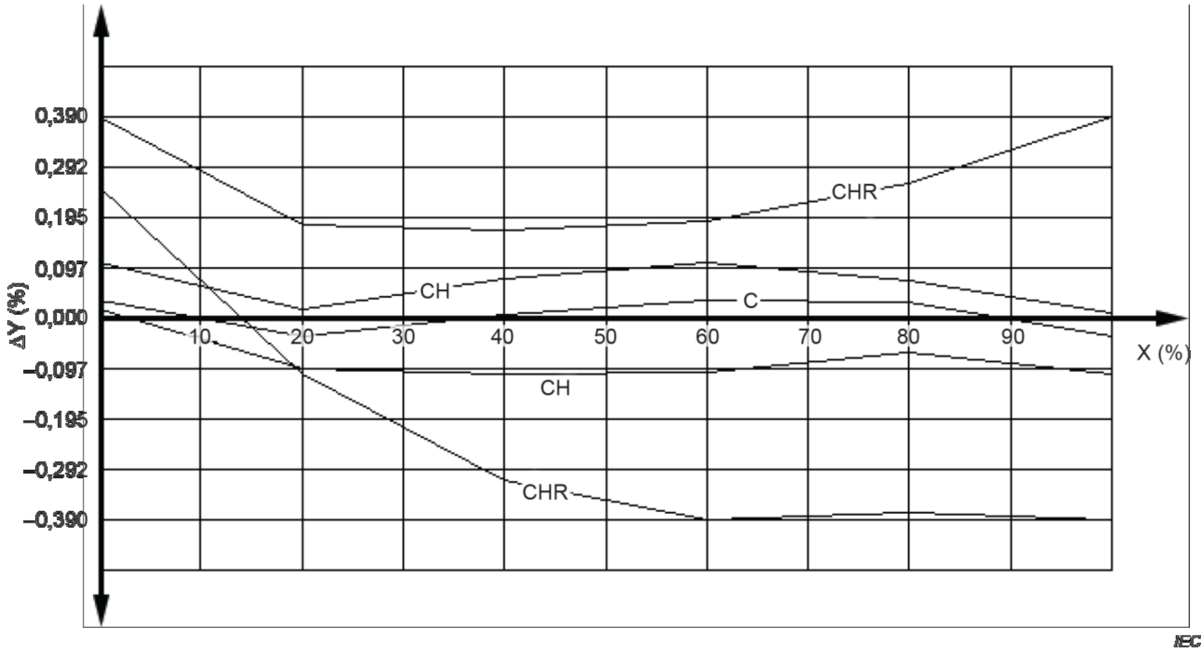
ξ_{CH} = ±0,109 % (calculated relative to the best reference curve)

ξ_{CH} = 0,365 % (calculated relative to the best working curve)

The total uncertainty:

(conformity plus hysteresis plus repeatability, and

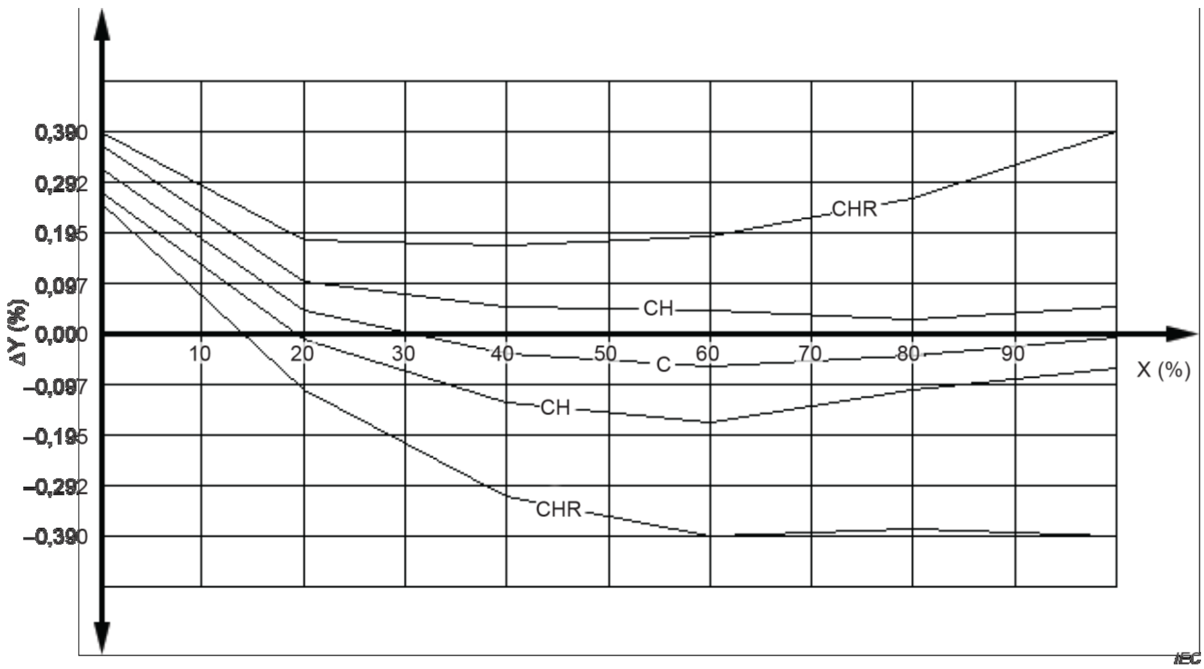
ξ_{CHR} = ±0,390 % it is calculated with the best working curve of the second degree and the actual uncertainty zone)



C –– Conformity; CH –– Conformity plus Hysteresis

CHR –– Conformity plus Hysteresis plus Repeatability

Figure C.3 – Deviation curves which are calculated relative to relevant best reference lines of the second degree



C –– Conformity; H –– Conformity plus Hysteresis

CHR –– Conformity plus Hysteresis plus Repeatability

Figure C.4 – Deviation curves which are calculated relative to the working line of the second degree

C.2.3 Numerical example 3

The original data is the same as that in numerical example 1. In this example, the outputs of the transducer remain unchanged, but the inputs (x) are amplified by a factor of 100. Suppose the transducer has a working equation of the form: Y = x. That is to say that this transducer is regarded as a transducer with a digital display.

The calculation process is omitted for brevity. The calculation results are listed below. In this case the full-span output is Y_{FS} = 1000, a little different from that in numerical example 1, so some performance indicators have values which are also a little different from their counterparts in numerical example 1.

Linearity:

ξ_L = −3,484 % (calculated in relation to the given working straight line)

ξ_L = ±0,167 % (calculated in relation to the best reference straight line)

Hysteresis:

$$\xi_H = 0{,}206\ \%$$

Linearity plus hysteresis:

ξ_{LH} = −3,542 % (calculated in relation to the given working straight line)

ξ_{LH} = ±0,239 % (calculated in relation to the best reference straight line)

Repeatability:

ξ_R = 0,325 % (coverage factor c = $t_{0,95}$ = 2,776)

The total uncertainty: (conformity plus hysteresis plus repeatability)

ξ_{LHR} = −3,855 % (calculated with the given straight line and the actual uncertainty zone)

ξ_{LHR} = ±0,443 % (calculated with the best reference straight line and the actual uncertainty zone)

The equation of the best reference straight line:

$$Y = -2{,}4445 + 0{,}9672x$$

NOTE It is obvious that, with ξLHR = −3,855 %, this transducer is not satisfactory as a digital indicator. However, with ξLHR = ±0,443 % which is calculated with the best reference straight line and the actual uncertainty zone, it has the potential to become a very good digital indicator, if the transducer has physical adjustment means, which can be used to make the given straight line to become a best straight line.

Annex D
(informative)

Examples for calculating transmitter individual and combined performance indicators

D.1 General principles of calculation

The calculation of transmitter individual and combined performance indicators is the same as that for transducers. The only difference lies in that the working straight line for transmitters is given in advance.

D.2 Numerical example

Table D.1 shows the original calibration data of a certain transmitter. The data, according to our inspection, are free from suspect and unreasonable data points. The given working equation is: Y = 2,0000 + 0,8000x. Calculate the individual and combined performance indicators of the transmitter.

Table D.1 – The original data obtained in the calibration

Travel	**Input** (x)	**Transmitter output** (y)				
		y_1	y_2	y_3	Y_4	Y_5
↓ Up	0,0	1,9995	1,9994	1,9996	1,9993	1,9994
	2,0	3,5966	3,5968	3,5963	3,5966	3,5967
	4,0	5,1942	5,1945	5,1941	5,1944	5,1946
	6,0	6,7957	6,7955	6,7953	6,7956	6,7956
	8,0	8,3995	8,3992	8,3993	8,3998	8,3994
	10,0	9,9987	9,9989	9,9986	9,9987	9,9989
Down ↑	10,0	9,9995	9,9991	9,9993	9,9992	9,9995
	8,0	8,3996	8,4000	8,3995	8,3999	8,3997
	6,0	6,7960	6,7962	6,7959	6,7963	6,7962
	4,0	5,1943	5,1947	5,1944	5,1946	5,1948
	2,0	3,5966	3,5969	3,5966	3,5967	3,5968
	0,0	1,9997	1,9996	1,9999	1,9997	1,9995

D.3 Calculation results

The calculation process is omitted for brevity. The calculation results are listed below. The deviation curves in this example are shown in Figure D.1 and Figure D.2.

Linearity:

ξ_L = −0,0692 % (calculated relative to the given working straight line)

ξ_L = ±0,0320 % (calculated relative to the best reference straight line)

Hysteresis:

$$\xi_H = 0,0072\ \%$$

Linearity plus hysteresis:

$$\xi_{LH} = -0,0705\ \%\ \text{(calculated relative to the given working straight line)}$$

$$\xi_{LH} = \pm 0,0334\ \%\ \text{(calculated relative to the best reference straight line)}$$

Repeatability:

$$\xi_R = 0,0080\ \%\ \text{(coverage factor } c = t_{0,95} = 2,776\text{)}$$

The total uncertainty (conformity plus hysteresis plus repeatability):

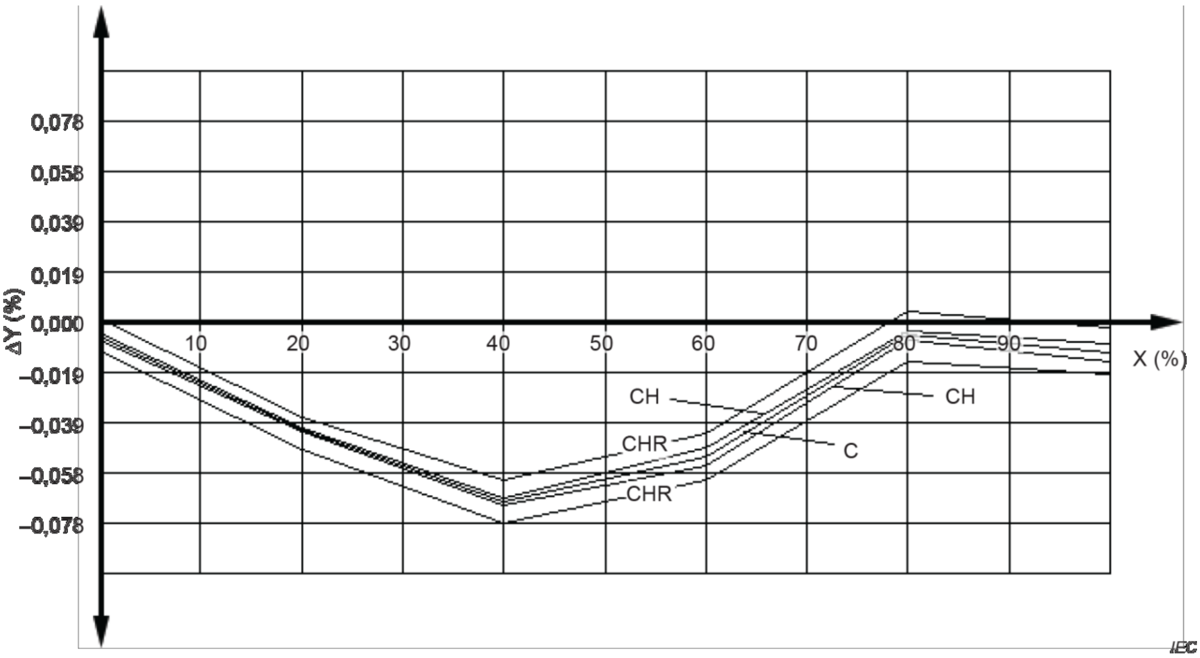
$$\xi_{LHR} = -0,0777\ \%\quad \text{(calculated with the given straight line and the actual uncertainty zone)}$$

$$\xi_{LHR} = \pm 0,0401\ \%\quad \text{(calculated with the best reference straight line and the actual uncertainty zone)}$$

The equation of the best reference straight line:

$$Y = 1,9969 + 0,8000x$$

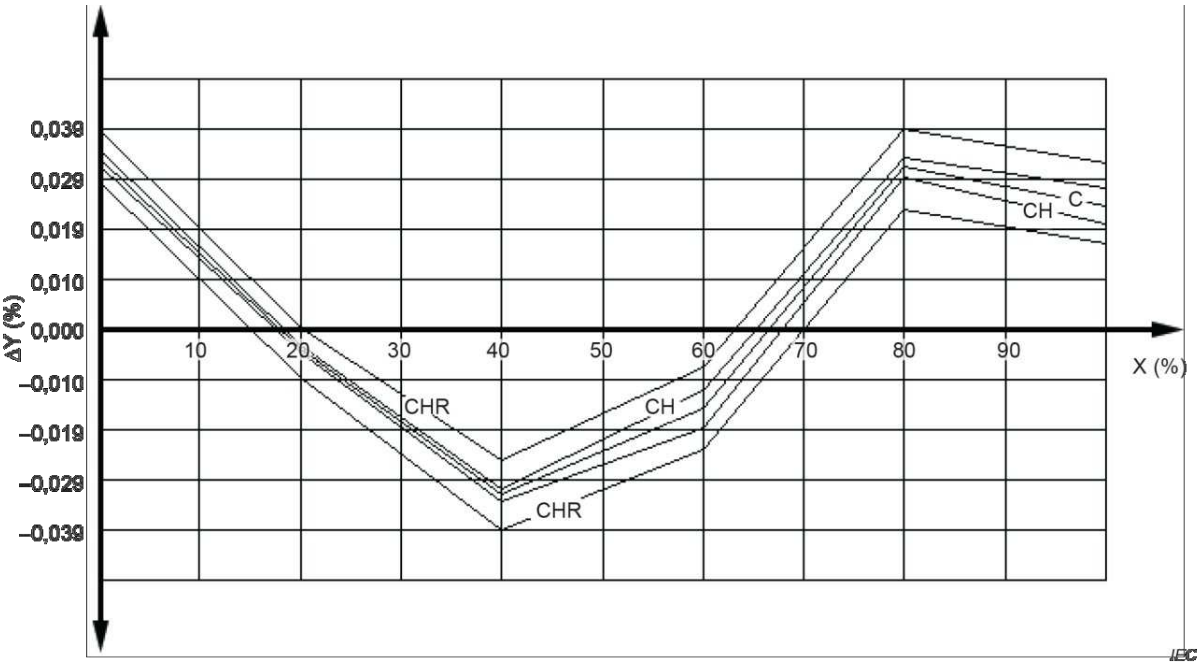
NOTE It is evident that the total uncertainty calculated with the best reference straight line is much better than that calculated with the given working straight line. If the transmitter has physical adjustment means, the given straight line can be made to become a best straight line.



C —— Conformity; CH —— Conformity plus Hysteresis

CHR —— Conformity plus Hysteresis plus Repeatability

Figure D.1 – Deviation curves which are calculated relative to the given working straight line



C ── Conformity; CH ── Conformity plus Hysteresis

CHR ── Conformity plus Hysteresis plus Repeatability

Figure D.2 – Deviation curves which are calculated relative to the best reference straight line

Annex E
(informative)

The Pre-treatment of the Original Data

E.1 The discovery of suspect and unreasonable data points

Suspect and unreasonable data points may occur in any measurements. The reasons for this are very complicated: the temporary, random or unpredictable troubles of the device under calibration, of the environmental conditions, or even of the calibrating devices themselves. The original data which contain suspect or unreasonable data points cannot be accepted as the real characteristics of the device under calibration and therefore cannot be used to make performance calculation.

It is not advisable to discard the suspect or unreasonable data points by statistical methods. The correct altitude toward dealing with them is: neither to accept nor to discard them rashly. Instead, try hard to discover them and find the real reasons for their existence remove the troubles and start a new calibration test again for trouble-free original data.

E.2 The detection of suspect data points

E.2.1 The general principle of statistical detection

In order to check whether the jth data point at the ith calibration point in the up-travel is a suspect data point, first calculate $\overline{y}_u$(i) and $s_{u,i}$. If the following discriminant is satisfied:

$$\text{Max} \mid y_u(i,j) - \overline{y}_u(i) \mid > ks_{u,i} \tag{E.1}$$

$$(i = 1\sim m;\ j = 1\sim n)$$

where

m is the number of calibration points;

n is the number of calibration cycles.

Then, this y_u(i,j) is regarded as a suspect data point. In Equation (E.1), k is the confidence factor that depends on the confidence level required (usually 95 %) and the sample size n (namely the cycle number n), to be checked. What should be checked is all the 2m × n data point in 2m samples in all the up-travel and down-travel cycles. If we find y_u(i,j) to be a suspect data point, in order to determine if there remains another suspect data point, the following two methods may be helpful:

1) Discard y_u(i,j) from the sample, then calculate $\overline{y}_u$(i) and $s_{u,i}$ again with the remaining data points, and make a new check of the remaining data points with Equation (E.1). In this method, every time with sample size becoming smaller and smaller, accurate check becomes more difficult.
2) Let $y_u(i,j) = \overline{y}_u(i)$, then calculate $\overline{y}_u$(i) and $s_{u,i}$ again with the slightly-modified sample, and make a new check of the sample with Equation (E.1). In this method, the sample size remains the same, so the check generally keeps being effective.

E.2.1.1 Detection method by Grubbs

This method was first put forward in 1950 by F.E. Grubbs. Being more effective and accurate, it has found wide application in many publications. The way it is used is the same as that in E.2.1. Table E.1 shows the relationship of its k to n.

Table E.1

n	3	4	5	6	7	8	9	10
k	1,153	1,463	1,672	1,822	1,938	2,032	2,110	2,176

E.2.1.2 Detection Method by AEDC

This method was first put forward in 1950 by Arnold Engineering Development Center of the American Air Force [07]. The concept of this method is very similar to that of Grubbs method. The only difference lies in that, with the increasing sample size, k becomes a little smaller than its counterpart in Grubbs method.

AEDC method is reported as being more accurate in checking small samples for suspect data points. The way it is used is the same as that in E.2.1. Table E.2 shows the relationship of its k to n.

Table E.2

n	3	4	5	6	7	8	9	10
k	1,154	1,435	1,634	1,782	1,896	1,988	2,064	2,127

E.2.1.3 Panorama-visual Inspection Method

Statistical methods are not all-powerful at all times. It is almost impossible to check a sample of less than 3 elements with statistical methods. Even a sample with 5 elements is still regarded as not being large enough for statistical treatment.

It is our common knowledge that under good calibrating conditions, there can be no abrupt and unreasonable changes of any kinds appearing in a device under test with stable performance. In light of this fact, it may be a good idea to discover the abrupt and unreasonable changes with the help of a full-view deviations graph. In most transducer and transmitter applications where the sample size is usually 3~5, sometimes the panorama-visual inspection method may probably become our only or last resort with which to discover suspect data points.

As some high-precision devices under test may have very good repeatability, for example better than 0,01 %, even the existing high-precision measuring instruments and the very careful calibration operations are not competent for their effective and accurate calibration. In these cases some of the good data points are easily checked as suspect ones, or vice versa. It is suggested that panorama-visual inspection method should be used in these circumstances. If the total picture or the general tendency of the deviation curves are good, then we can make decision that the device under test should be allowed to pass the check, even if there are some statistically-found suspect data points, or vice versa.

E.2.1.4 Numerical Example for Discovering Suspect Data Points

The original calibration data of a certain transducer is shown in Table E.3, make a check of them for suspect data points with Grubbs detection method, AEDC detection method and the panorama-visual inspection method.

Table E.3 – The original data obtained in the calibration

travel	Input (x)	Transducer output (y)				
		y_1	Y_2	y_3	y_4	Y_5
↓ Up	0,0	-2,774	-2,714	-2,681	-2,672	-2,663
	2,0	0,560	0,610	0,637	0,652	0,657
	4,0	3,945	3,987	4,022	4,037	4,042
	6,0	7,385	7,422	7,451	7,474	7,470
	8,0	10,875	10,920	10,940	10,944	10,964
	10,0	14,420	14,467	14,464	14,478	14,492
Do wn ↑	10,0	14,420	14,467	14,464	4,478	14,492
	8,0	10,944	10,881	10,985	10,985	11,011
	6,0	7,489	7,517	7,518	7,550	7,551
	4,0	4,052	4,090	4,107	4,106	4,122
	2,0	0,655	0,680	0,698	0,708	0,721
	0,0	-2,714	-2,681	-2,664	-2,651	-2,640

E.2.1.4.1 By Using Grubbs Detection Method

Conclusion: No suspect data point is discovered.

E.2.1.4.2 By Using AEDC Detection Method

Conclusion: The two data points $y_u(6,1)$ and $y_d(6,1)$ are discovered as suspect data points, because we know:

$$y_u(6,1) = y_d(6,1) = 14,420;$$

$$\overline{y}_u(6) = \overline{y}_d(6) = 14,4642;$$

$$s_{u,6} = s_{d,6} = 0,02704;\ k = 1,634$$

Therefore,

$$|\ y_u(6,1) - \overline{y}_u(6)\ | = 0,04420 > ks_{u,5} = 0,04418;$$

$$|\ y_d(6,1) - \overline{y}_d(6)\ | = 0,04420 > ks_{u,5} = 0,04418$$

From the above calculations, we know, even though yu(6,1) and yd(6,1) are discovered as suspect data points, but at most they are the edge-lying suspect data points. This is somewhat a dilemma in which discarding or accepting the two points is hard for us to decide. In this case, we might have a try of the panorama-visual inspection method. From Figure E.1, we can see that there is no abrupt change in the deviation curves at the two data points. So they can be accepted as good data points.

E.2.1.4.3 By Using Panorama-visual Inspection Method

From Figure E.1, at x = 0.8 in the down-travel, we find that there is a small upward turn in the CHR deviation curve. We also find that this data point $y_d(5,2)$ = 10.881 deviates obviously from the majority. But strangely, AEDC method shows that it is a good data point. The AEDC checking process goes like this:

y_d (5,2) = 10,881;

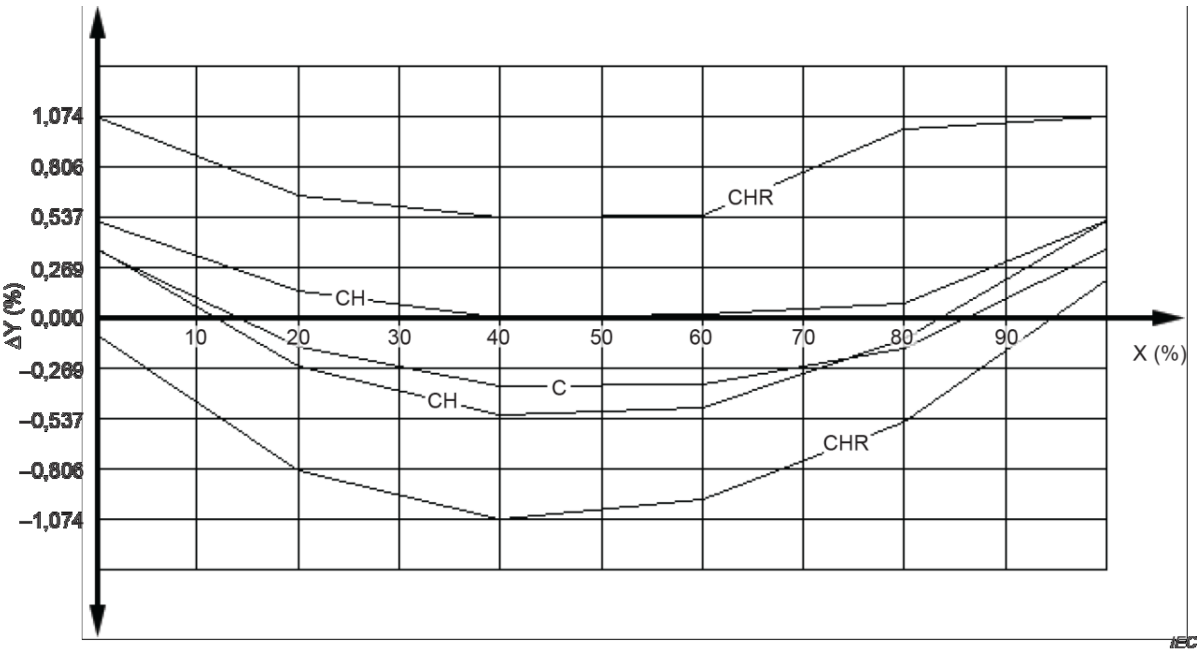
$\overline{y}_d$ (5) = 10,9612;

$s_{d,5}$ = 0,05084; k = 1,634

Therefore,

$| y_d(5,2) - \overline{y}_d (5) | = 0,08020 < k_{su,5} = 0,08307$

This is also a contradictory case. At least, we believe this data point an edge-lying good data point. In this case, it is desirable to find the real reason for the upward turn of the CHR deviation curve and take substantial measures to remove the possibly hidden troubles.



C —— Conformity; CH —— Conformity plus Hysteresis

CHR —— Conformity plus Hysteresis plus Repeatability

Figure E.1 – Deviation curves which are calculated relative to the best working straight line

E.3 The Inspection of Unreasonable Data Points

E.3.1 The Unreasonable Data Points

There are some original data which can or roughly can pass all the statistical detections, and yet they are questionable in reality. These data can be classified as unreasonable data and they may appear in the following different ways:

1) A group of data taken at the same calibration point and in the same calibration direction show an evidently gradually-increasing or gradually-decreasing tendency in relation to the successive cycles. This tendency can be discovered by the comparison of a group of the adjacent-in-cycle data pairs, for example y_u(i,j) and y_u(i,j+1). This kind of unreasonable

data result mainly from the change in environmental temperature or from the self-heating of the device under calibration.

2) At the upper-range-value the hysteresis becomes zero. Generally, this is not possible. Because the correct testing operation demands that at the upper-range-value, the measurand should have a little over-shoot before starting the down-travel.

3) All or some of the hysteresis values become negative. Generally, this is also not possible. The reasons for this abnormality may be temperature change, range switching of measuring instruments, un-stable or poor-quality calibration facilities or careless operation. This may also result from the poor quality of the device under calibration.

4) The appearance of abrupt changes in repeatability value or even the appearance of zero repeatability. The reasons for this may be more than those mentioned above.

E.3.2 Example 1 for Inspecting the Unreasonable Data Points

The results of a computer-conducted inspection for the unreasonable data points are shown in Table E.4. This inspection method is more qualitative than quantitative. The following is a brief analysis of the inspection results.

Table E.4 – A list of the computer-conducted inspection results for the unreasonable data points

Items to be inspected	Percentage of the gradually-increasing adjacent-in-cycle data pairs	Percentage of the gradually-decreasing adjacent-in-cycle data pairs	Percentage of the equal adjacent-in-cycle data pairs	H = 0	H becomes negative
Data in Table C.1	50,00 %	50,00 %	0,00 %	0,0 %	0,0 %
Data in Table D.1	56,25 %	41,67 %	2,08 %	0,0 %	0,0 %
Data in Table E.3	87,50 %	10,42 %	2,08 %	100 %	3,3 %

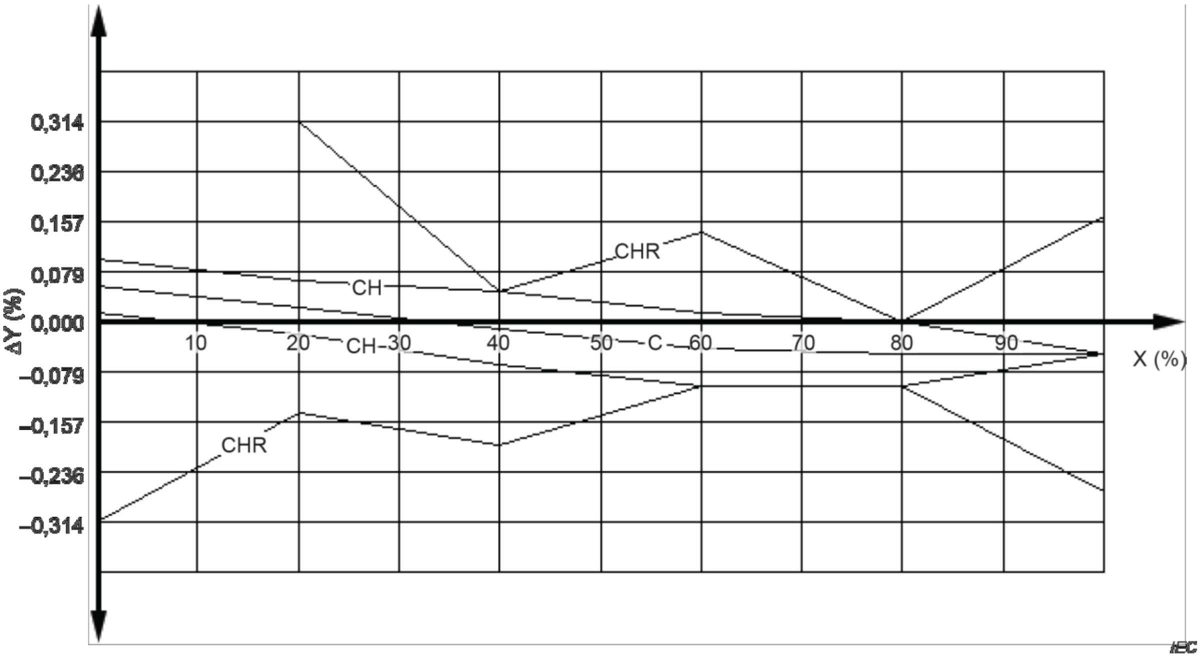
1) From Table E.4, we find that there are, on the main, no unreasonable data points in Table C.1 and Table D.1. The results of Table C.1 even show a typical instance of good original calibration data.

2) The data in E3 show a general tendency of considerable increase with cycles, which may result from temperature change.

E.3.3 Example 2 for Inspecting the Unreasonable Data Points

Figure E.2 shows the deviation curves of a certain transducer (m = 6, n = 3, the original calibration data are omitted). For so small a cycle number, the statistical detection methods cannot make an accurate check for suspect data points.

From Figure E.2, we can find troubles with the original data: zero hysteresis and zero repeatability, and very obviously abrupt and irregular change in repeatability. It is absolutely certain, that this original data is very typical of a poor calibration operation or of a poor device under calibration.



C ── Conformity; H ── Conformity plus Hysteresis

CHR ── Conformity plus Hysteresis plus Repeatability

Figure E.2 – Deviation curves which are calculated relative to the best working straight line

Annex F
(informative)

The fundamentals for calculating transducer uncertainty

F.1 Components of measurement uncertainty

Uncertainty is an evaluation that shows a zone in which the true value of a measurand lies under specified operating conditions, whereas error is the deviation of the measured value of a measurand from its true value. In general, uncertainty is composed of several components:

$s_1, s_2, \ldots, s_i, \ldots$ (components of A class, obtained statistically)

$b_1, b_2, \ldots, b_j, \ldots$ (components of B class, obtained non-statistically)

F.2 Combined uncertainty

Combined uncertainty is calculated by using Equation (F.1):

$$u_c = \sqrt{\sum s_i^2 + b_i^2 + 2\sum_{k<1} \mathrm{cov}(\sigma_k, \sigma_l)} \quad \text{(F.1)}$$

where

$\mathrm{cov}(\sigma_k, \sigma_l)$ is the covariance of any two components σ_k, σ_l (which may be regarded as any one of s_l and b_j). This covariance can be also expressed in terms of correlation coefficient ρ_{kl}, namely: $\mathrm{cov}(\sigma_k, \sigma_l) = \rho_{kl}\sigma_k\sigma_l$.

F.3 The combined uncertainty of a transducer

For a single-input/single-output transducer, at the ith calibration point, the total random error si may be regarded as the only component of class A, whereas the total systematic error bI may be regarded as the only component of class B. Therefore at the ith calibration point, the combined uncertainty of the transducer is:

$$U_{c,i} = \sqrt{b_i^2 + s_i^2 + 2\rho_{i,(bs)}bs} \quad \text{(F.2)}$$

where

$\rho_{i,(bs)}$ The correlation coefficient of b to s at the ith calibration point.

In transducer performance calculation, the random error and the systematic error are calculated from the same original data, so b and s are well correlated to each other. Therefore we consider that ρi,(bs) = 1. Thus the combined uncertainty of the transducer at the ith calibration point can be reduced to the simple form:

$$U_{c,i} = b_i + s_i \quad \text{(F.3)}$$

F.4 The total uncertainty of a transducer at the ith calibration point

This is also called the expanded uncertainty. In the case of t-distribution and with a sample size of 3~5, the confidence level of combined uncertainty is about 60 %~70 %, which is not satisfactory in most applications. In order to achieve a feasible total uncertainty at the ith calibration point $U_{t,i}$, the combined uncertainty is usually multiplied by a so-called coverage factor c:

$$U_{t,i} = cu_{c,i} = cb_i + cs_i \quad (F.4)$$

where

C is Coverage factor. For t-distribution and for a confidence level of 95 %, as we do in 4.7.2, c = t 0.95;

cb_i is Let $cb_i = B_i$. B_i is called the extreme value of the total systematic error of a transducer at the ith calibration point, which can usually be obtained by non-statistical methods;

cs_i is The extreme value of the total random error of a transducer at the ith calibration point. And we have csI = t0.95sI.

Therefore finally, the total uncertainty of a transducer at the ith calibration point will take the following equation form:

$$U_{t,i} = \pm(B_i + t\ 0.95 s_i) \quad (F.5)$$

F.5 The total uncertainty of a transducer

The total uncertainty of a transducer at the ith calibration point, when the largest one of which is chosen and expressed as the percentage of transducer full-span output, turns into the total uncertainty of a transducer, and it takes the following equation form:

$$U_t = \pm \frac{\mathrm{Max}\left|U_{t,i}\right|}{Y_{FS}} \times 100\,\% \quad (F.6)$$

$$U_t = \pm \frac{\mathrm{Max}\left|B_i + t\ 0{,}95\ s_i\right|}{Y_{FS}} \times 100\,\% \quad (F.7)$$

where

B_i is the extreme value of the total systematic error of a transducer at the ith calibration point, which can usually be obtained by non-statistical methods;

$t\ 0{,}95 s_i$ is the extreme value of the total random error of a transducer at the ith calibration point.

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